CHAPTER 3

Laws of Motion

Topic Covered

- Aristotle's Fallacy - Force
- The Law of Inertia
- Newton's 1st Law of Motion
- Newton's 2nd Law of Motion
- Newton's Third Law of Motion
- Applications of Newton's Laws
- Law of Conservation of momentum
- Equilibrium of a Particles
- Common Force in Mechanics - Friction
- Dynamics of Circular Motion

INTRODUCTION

In the previous chapter, we were only concerned about the motion of a particle in space quantitatively. We saw that uniform motion needs the concept of velocity alone whereas non-uniform motion requires the concept of acceleration in addition. Now the question arises who governs the motion of the bodies. We know that in order to move a football from rest, someone must kick it. To throw a stone upwards, one has to give it an upward push. A breeze causes the branches of a tree to swing. A boat moves in a flowing river without anyone rowing it. Clearly, some external agency is needed to provide force to move a body from rest.

ARISTOTLE'S FALLACY

A **fallacy** is an argument which provides poor reasoning in support of its conclusion. An argument can be fallacious whether or not its conclusion is true. The Greek thinker, **Aristotle** held the view that if a body is moving, **something external** or external agency is required to keep in motion. The Aristotelian law of motion may be phrased for our purpose here, **An external force is required to keep a body in motion**.

FORCE

A **force** is a push or a pull. A force given to an object provides energy to it causing the object to start moving, stop moving, change its speed, change its shape or change its direction.

Or it is an external effort in the form of push or pull, which

(i) Produces or tries to produce motion in a body at rest, or

(ii) Stops or tries to stop a moving body, or

(iii) Changes or tries to change the direction of motion of the body.

The following factor determine the effect that a force has on a body

(i) The magnitude of force applied

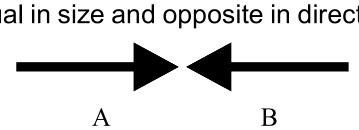
(ii) The direction in which force is applied

(iii) The point of application of force.

Forces always occur in pair, it can be either balanced or unbalanced.

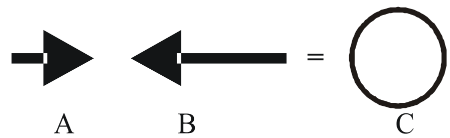
BALANCED FORCES

Balanced forces are those forces which doesn't cause change in motion. They are equal in size and opposite in direction.



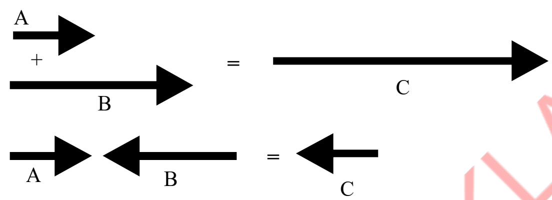
As the force that each of them are exerting is equal, the two forces cancel each other out and the resulting force is zero. Therefore, there is no change in motion.

Tug of war is a good example to see balanced forces in action.



UNBALANCED FORCES

Unbalanced forces always cause a change in motion. They are not equal and opposite. When two unbalanced forces are exerted in opposite directions, their combined effect/force is equal to the difference between the two forces and is exerted in the direction of the larger force. Look at the following examples to help make this more clearer.



FUNDAMENTAL FORCES OF NATURE

Different forces in nature -

(A) Gravitational force or interaction

(B) Electromagnetic force or interaction

(C) Nuclear force or interaction,

(D) Weak force or interaction,

(A) Gravitational force

This force acts in between two masses.

The nature of this force is attractive.

This force can be obtained from Newton's gravitational law.

According to Newton's gravitational law

$F \propto m_1 m_2$

$F \propto 1/r^2$

or $F = G\, m_1 m_2 / r^2$

In vector notation, the force acting on m_2 due to m_1

$$\vec{F}_{21} = G\frac{m_1 m_2}{|\vec{r}_{21}|^2}\hat{r}$$

G is constant which is called Gravitational constant. It is a universal constant. Its value is

$G = 6.67 \times 10^{-11}$ N–m^2 /kg^2

and dimensions are $M^{-1}L^{3}T^{-2}$

The range of this force is very large.

This force acts in between the planets, here its magnitude is very large. This force also acts in between atomic particles (e – e), (e – p), (p – p), (p – n) but in this case its magnitude is very small. This force is weakest amount all forces in nature. Energy particle of this force is **Graviton.**

- This force is important in those circumstances in which one body taking part in the interaction is having astronomical size.
- Gravitational force of 6.67×10^{-11} newton acts between two bodies of mass 1kg each placed at a distance 1 m apart.
- The interaction time of this force is about 10^{-2} sec.
- A force of 2.3×10^{20} newton acts in between the moon and the earth whereas a force of 4.054×10^{-47} newton acts in between an electron and a proton of hydrogen atom.

(B) Electromagnetic force

This force acts between charged or magnetized objects. The force acting between stationary or moving charges is electromagnetic force. Source of electrical forces are stationary and moving charges. Magnetic force acts only on moving charges. It can be attractive or repulsive.

There is repulsive force between the charges of similar nature and attractive force between the charges of dissimilar nature.

Coulomb's law

$F \propto q_1 q_2$

$F \propto 1/r^2$

or

$F = K q_1 q_2 / r^2$

where K is a constant which depends upon the nature of the medium. For vaccum

$$K = \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \, N - m^2 / C^2$$

ϵ_0 is called permittivity of vacuum and its value is $8.85 \times 10^{-12}\ C^2/N - m^2$

The range of this force is also very large.

Electromagnetic force is 10^{36} times stronger than the gravitation force but $\frac{1}{137}$ times weaker than the nuclear force.

(C) Nuclear force

- This force acts within the nucleus of an atom.
- This force acts only among protons and neutrons within the nucleus of an atom.
- Nature of this force in unknown and it has unique properties; therefore it is also called strange force.
- This force is maximum inside the nucleus and zero outside the nucleus.
- The range of the force is very small ($\sim 10^{-15}$ m)
- This force is strongest force of nature
- Its energy particle is **pion**.
- The magnitude of this force can be obtained from Yukawa potential:

$$U(r) = U = U_0 \frac{e^{-r/r_0}}{r}$$

where U_0 and r_0 are constants.

Nuclear force does not depend on charge i.e., it is charge independent. This force acts between proton-proton, neutron-neutron and neutron-proton in equal strength.

(D) Weak forces -

- β decay in radioactive disintegration is explained by this force.
- This force is weaker in comparison to nuclear and electromagnetic forces and is 10^{-25} times the nuclear force.

  The range of this force is of the order of 10^{-15} m . The energy particle of this force is **boson**.

  Neutron and proton in the nucleus decay due to weak forces as

  (Antineutrino) ${}_1n^0 \rightarrow {}_1p^1 + \beta^{-1} + \overline{\nu}$

  (Neutrino) ${}_1p^1 \rightarrow {}_1n^0 + \beta^{+1} + \nu$

  This force is related with the transformation of neutron to proton or proton to neutron in β decay.

  The nature of weak forces is still unknown.

Comparative Study of Forces

S.No.	Forces	Nature	Range	Relative strength	Energy particle
1.	Gravitational	attractive	infinite	1	graviton
2.	Electromagnetic	attractive or repulsive	infinite	10^{36}	photon
3.	Nuclear	attractive	very short	10^{39}	pion
4.	Weak	unknown	very short	10^{14}	boson

TYPES OF FORCES

Field Forces

Those forces which do not require contact between the bodies to act, for example gravitational force, electro-magnetic force etc., are known as field forces.

Contact Force (F)

Forces which act when bodies are in contact are known as contact forces. It is usually convenient to resolve contact forces into components, one parallel to the surface of contact, the other perpendicular to the surface of contact.

Normal Force or Normal Reaction (N):

The component of the contact force that is perpendicular to the surface of contact is known as the Normal reaction force. It measures how strongly the surfaces in contact are pressed together.

Frictional Force (f):

It is the component of the contact force parallel to the surface of contact. The direction of the force of friction is opposite to the direction of relative motion between the surfaces, or is such as to oppose any tendency of relative motion between the surfaces.

Tension:

It is inter molecular force between the atoms of a string, which acts or reacts when the string is stretched. There are some important points regarding the tension in a string.

(i) Force of tension act on a body in the direction away from, the point of contact or tied ends of the string.

(ii) If a string is mass less, the tension in it is same everywhere; on the other hand if a string is not mass less (i.e. its mass is not negligible), tension at different points may be different.

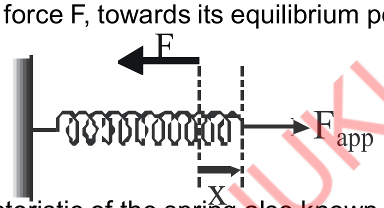
(iii) If friction is present between the pulley and the string, the tension in the string will be more on the side of larger mass.

SPRING FORCE

An ideal spring follows Hooke's law which says that the force applied by a spring on bodies connected to it is proportional to its extension or compression (change in length from its natural length).

We know that the more force we apply to spring the more it stretches i.e. the extension of the spring is proportional to the applied force. Figure shows a spring in its equilibrium length. If we stretch is by a distance x from its equilibrium position it applies a force F, towards its equilibrium position (Restoring Force) which is proportional to x

$F \propto -x$



$F = -kx$

where k is a constant that is characteristic of the spring also known as the spring constant.

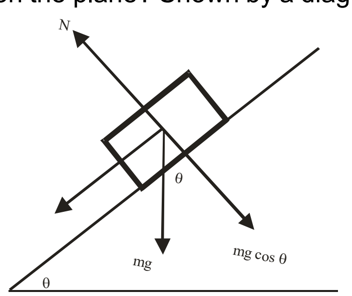
(–) sign indicate nature of force restoring. A spring has tendency of restoring its equilibrium position, thus whether we stretch it or compress, and it always opposes the external force in the direction towards its equilibrium position.

The forces exerted by a light spring on bodies connected to it at opposite ends are equal and opposite.

Spring constant of a spring is inversely proportional to its relaxed length $k \propto \frac{1}{l}$.

SOLVED EXAMPLE

Example 1. What is the magnitude of the force exerted by a smooth inclined plane of inclination θ, on a mass m placed on the plane? Shown by a diagram.



Solution. $mg\ \cos\theta$ $(N = mg\ \cos\theta$ in equilibrium)

Example 2. What is the magnitude & direction of force due to gravity?

Solution. mg, vertically downward.

Example 3. A car of mass 1000 kg is moving with a speed of 36 km/h on a level road. Calculate the retarding force required to stop the car in a distance of 50 m.

Solution. Here m =1000 kg ; u = 36 km/h = 10 m/s ; v = 0 ; S = 50 m

$$v^2 - u^2 = 2aS$$

$$a = \frac{0-100}{2x50} = -1ms^{-2}$$

$$F = ma = 1000(-1) = -1000N$$

Example 4. A Diwali rocket is ejecting 0.05 kg of gases per second at a speed of 400 m/s. What is the accelerating force on the rocket?

Solution. $F = ma = m\left(\frac{v-u}{t}\right) = m\left(\frac{v-0}{t}\right) = \frac{m}{t}v = 0.05\,x\,400 = 20\,N$

Example 5. A 600 kg rocket is set for a vertical firing. If the exhaust speed is 1000m/s , then what is the amount of gases ejected per second to supply the thrust needed to overcome the weight of the rocket? (g =10 ms^{-2})

Solution. $F = mg = 600\,x\,10 = 6000\,N$

$$F = ma = m\left(\frac{v-u}{t}\right) = m\left(\frac{v}{t}\right) = \left(\frac{m}{t}\right)v$$

$$(u = 0, \quad v = 1000)$$

$$\frac{m}{t} = \frac{F}{u} = \frac{6000}{1000} = 6\ kg/s$$

EXERCISE

1. If a bullet of mass 5 gm moving with velocity 100 m/sec, penetrates the wooden block upto 6 cm. Then the average force imposed by the bullet on the block is

(A) 8300 N (B) 417 N (C) 830 N (D) Zero

2. The mass of a lift is 2000 kg. When the tension in the supporting cable is 28000 N, then its acceleration is

(A) 30 ms^{-2} (B) 4 ms^{-2} upward (C) 4 ms^{-2} downward (D) 14 ms^{-2} upwards

3. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height furhter, find the magnitudwe of the force. Consider g = 10 m/s^2.

(A) 16 N (B) 20 N (C) 22 N (D) 4 N

THE LAW OF INERTIA

Inertia of a body refers to tandency of a body to resist any change in its state (of rest or of uniform motion in a straight line).

INERTIA OF REST

It is the tendency of a body to resist any change in its state of rest. This means a body which is at rest will remain at rest unless or until some external force is applied to it.

(A) If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.

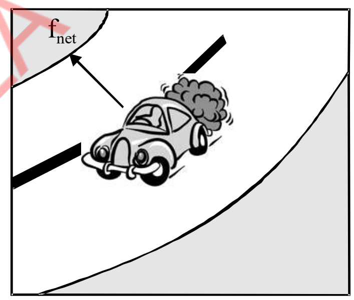
(B) The dust particles in a carpet fall off when it is beaten with a stick. This is because the beating sets the carpet in motion whereas the dust particles tend to remain at rest and hence get separated.

INERTIA OF MOTION

It is the tendency of a body to resist any change in its state of motion or it is the inability of a moving body to stop on its own. This means a body which is in motion will remain in motion along a straight line unless or until some external force is applied to it.

INERTIA OF DIRECTION

Inertia of direction:- The tendency of a body by which it resists a change in its direction of motion is called inertia of direction.



(i) When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

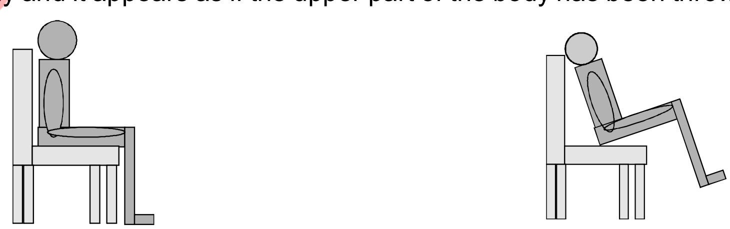
(ii) The rotating wheel of any vehicle thrown out 8, mud particle fly off tangentially due to directional inertia.

(iii) When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

SOLVED EXAMPLE

Example 1: Why a person who is standing freely in bus, thrown backward, when bus starts suddenly?

Solution. When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward.



Note :

(i) If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

(ii) When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.

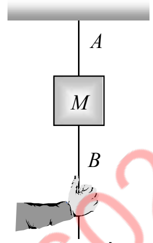
Example 2: In oil tankers some space is left at the top while filling them. Explain why?

Solution. When a moving oil tanker suddenly stops, the oil inside the tanker continues to be in the state of motion due to inertia of motion. As a result it splashes in the forward direction. similarly, when a stationary tanker suddenly sets into motion, the oil inside the tanker splashes in the backward direction. If no space is left at the top of the oil, it will overflow. Therefore, to prevent any overflow of the oil due to sudden start or stop of the tanker, some space is left vacant at the top while filling the tanker.

Example 3: In the arrangement shown in the figure, what will happen?

(A) If the string B is pulled with a sudden jerk?

(B) If the string B is pulled steadily the force applied to it?



Solution. (A) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.

(B) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break.

EXERCISE

1. Inertia is that property of a body by virtue of which the body is

(A) Unable to change by itself the state of rest

(B) Unable to change by itself the state of uniform motion

(C) Unable to change by itself the direction of motion

(D) Unable to change by itself the state of rest or of uniform linear motion

2. Two objects A and B have masses 50 kg and 60 kg. If their inertia of rest are I_A and I_B respectively then which one of the following is correct?

(A) $I_A > I_B$ (B) $I_A = I_B$ (C) $I_A < I_B$ (D) None of these

3. A motor cycle and a car are moving on a horizontal road with the same velocity. If they are brought to rest by the application of brakes, which provides equal retardation, then

(A) Motor cycle will stop at shorter distance

(B) Car will stop at a shorter distance

(C) Both will stop at the same distance

(D) Nothing can be predicted

NEWTON'S FIRST LAW OF MOTION

A body at rest will remain at rest unless or until some external force is applied on it. A body which is moving with uniform velocity will remain in that state of motion unless or until some external force is applied on it.

$F = ma$

if $F = 0$

$\Rightarrow$ a = 0 (m $\neq$ 0)

Examples:-

(i) We tend to fall forward when a bus suddenly stops. When bus stops suddenly, we tend to resist the change in our state of motion and hence fall forward.

(ii) We tend to fall backward when a bus suddenly starts.

(iii) We tend to get thrown to one side when a car takes a sharp turn.

SOLVED EXAMPLE

Example 1. A boy sitting on the topmost berth in the compartment of a train which is just going to stop on the railway station drops an apple aiming at the open hand of his brother situated vertically below his hands at a distance of about 2 m. The apple will fall

(A) In the hand of his brother

(B) Slightly away from the hand of his brother in the direction of motion of the train.

(C) Slightly away from the hands of his brother in the direction opposite to the direction of motion of the train.

(D) None of these

Solution. **(B)**

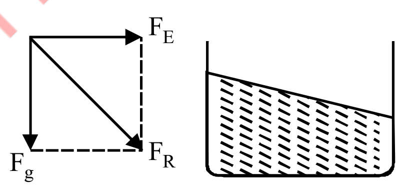
According to the Newton's first law of motion, the velocity of apple when dropped is equal to the velocity of train at that instant. As the velocity of train is decreasing but velocity of apple remains unchanged, the apple will fall ahead of his brother in the direction of motion of train.

Example 2. Drums of oil are carried in a truck. If a constant acceleration is applied on the truck, the surface of oil in drum will

(A) Remain unaffected (B) Rise towards forward direction

(C) Rise towards backward direction (D) Nothing is certain

Solution. **(C)**



When a constant acceleration is applied on the truck, a constant force acts on it. The resultant of this force and the force due to gravity act along the surface of the oil. Hence, the surface of oil rises towards backward direction.

EXERCISE

1. When a bus suddenly takes a turn, the passengers are thrown outwards because of

(A) Inertia of motion (B) Acceleration of motion

(C) Speed of motion (D) Both (B) and (C)

2. A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball fall

(A) Outside the car (B) In the car ahead of the person

(C) In the car to the side of the person (D) Exactly in the hand which threw it up

LINEAR MOMENTUM

- The product of mass of a body (m) and velocity ($\vec{v}$) is called linear momentum, i.e.,
- It is a vector quantity and its direction is in the direction of velocity.
- In SI system its unit is kg-m/s and dimensions are $M^1L^1T^{-1}$.
- If m_1 and v_1 are the mass and velocity of one body and m_2 and v_2 are the mass and velocity of another body, then

  $$\frac{p_1}{p_2} = \frac{m_1v_1}{m_2v_2}$$

  If $v_1 = v_2$ and $m_1 > m_2$

  then $p_1 > p_2$

  Thus the momentum of heavier body will be greater than the momentum of lighter body.
- If $p_1 = p_2$, then

  $m_1 v_2 = m_2 v_2$ or $\frac{v_1}{v_2} = \frac{m_2}{m_1}$

  If $m_1 > m_2$ then $v_2 > v_1$

  i.e. momentum being same, the velocity of heavier body will be lesser than the velocity of lighter body .

NEWTON'S SECOND LAW

(A) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

(2) If a body of mass m, moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow F = k\frac{d\vec{p}}{dt}$$ (k = 1 in C.G.S. and S.I. units)

or $$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \frac{m\vec{v} - m\vec{u}}{t_2 - t_1} = \frac{m(\vec{v} - \vec{u})}{t}$$

As $\vec{a} = \frac{(\vec{v} - \vec{u})}{t}$ (acceleration produced in the body)

$\therefore$ $\vec{F} = m\vec{a}$

Force = mass × acceleration

$\therefore$ $\vec{a} = \frac{\vec{F}}{m}$ or $\vec{a} \propto \frac{1}{m}$

Note : (A) First law can be obtained from second law.

(B) From this, law of conservation of linear momentum can be obtained, i.e.,

If F = 0, then dp/dt = 0

$\therefore$ p = mv = constant

(C) According to this law mass of a particle or body does not change with velocity

Example :

While catching a ball the cricketer moves hands backwards. In this case he needs to decrease the force with which ball is coming so that it doesn't hurt cricketer's hand, so he increases the time by lowering his hand and hence acceleration of the ball decreases.

– Use of springs and shockers in motor cars.

– A judo champ breaks a pile of bricks in one go. His action is too fast.

In this case he needs to increase the force with which he can break the pile, so he decreases the time by fastening his action.

$$F \alpha \frac{dp}{dt}$$

If we decrease t, F will increase

– In a high jump athlete event, the athletes fall on a cushioned bed to increase the time of stop. This results in the decrease of rate of change of momentum and hence force.

– High-speed photograph of a golf ball as it is struck by a club. The ball is termporarily deformed and is accelerated.

SOLVED EXAMPLE

Example 1. A train is moving with velocity 20 m/sec. on this, dust is falling at the rate of 50 kg/min. The extra force required to move this train with constant velocity will be

(A) 16.66 N (B) 1000 N

(C) 166.6 N (D) 1200 N

Solution : **(A)**

Force $F = v\frac{dm}{dt} = 20 \times \frac{50}{60} = 16.66\text{N}$

Example 2. A force of 10 Newton acts on a body of mass 20 kg for 10 seconds. Change in its momentum is

(A) 5 kg m/s (B) 100 kg m/s

(C) 200 kg m/s (D) 1000 kg m/s

Solution : **(B)**

Change in momentum = force × time = 10 × 10 = 100 kgm/sec

Example 3. A vehicle of 100 kg is moving with a velocity of 5 m/sec. To stop it in $\frac{1}{10}$ sec, the required force in opposite direction is

(A) 5000 N (B) 500 N

(C) 50 N (D) 1000 N

Solution : **(A)**

m = 100 kg, u = 5m/s, v = 0, t = 0.1 sec

$$\text{Force} = \frac{mdv}{dt} = \frac{m(v-u)}{t} = \frac{100(0-5)}{0.1}$$

F = – 5000 N

Example 4. A force of 5 N gives a mass M_1, an acceleration equal to 8 m/s^2 and M_2, an acceleration = 24 m/s^2. What is the acceleration if both masses are tied together?

(A) 16 m/s^2 (B) 6 m/s^2

(C) 12 m/s^2 (D) 4 m/s^2

Solution. **(B)**

Mass M_1 = 5/8 kg, Mass M_2 = 5/24 kg.

Total mass = M_1 + M_2 = 5/8 + 5/24 = 20/24 kg.

Therefore, total acceleration in two masses = F/(M_1 + M_2) = $\frac{5}{20/24} = \frac{5 \times 24}{20} = 6 ms^{-2}$

Example 5. Can a single isolated force exist in nature? Give reason.

Solution. No, single isolated force cannot exist in nature. For every action there is equal and opposite reaction. So forces always appear in pairs.

Example 6. A body of mass m moves along X axis such that its position co-ordinate at any instant t is $x = at^4 - bt^3 + ct$. Where a, b, c are constant. What is the force acting on the particle at any instant?

Solution. Position co-ordinates, $X = at^4 - bt^3 + ct$

Velocity $\frac{dx}{dt} = 4at^3 - 3bt^2 + c$

Acceleration = $\frac{d^2x}{dt^2} = \frac{d}{dt}(4at^3 - 3bt^2 + c) = 12at^2 - 6bt$

Force = mass x acceleration = $m(12at^2 - 6bt)$

Example 7. Express Newton's second low of motion in component form. Give its significance.

Solution. Force, momentum and acceleration can be represented in their rectangular components.

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{K}$$

$$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{K}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{K}$$

The vector form of second law of motion.

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

$$F_x\hat{i} + f_y\hat{j} + F_zK = \frac{d}{dt}(P_x\hat{i} + P_y\hat{j} + P_z\hat{k})$$

$$= m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$$

$$F_x = \frac{dP_x}{dt} = ma_x$$

$$F_y = \frac{dP_y}{dt} = ma_y$$

$$F_z = \frac{dP_z}{dt} = ma_z$$

EXERCISE

1. A player stops a football weighing 0.5 kg which flies towards him with a velocity of 10m/s. If the impact lasts for 1/50th second and the ball bounces back with a velocity of 15 m/s, then the average force involved is

(A) 250 N
(B) 1250 N
(C) 500 N
(D) 625 N

2. We can derive Newton's

(A) Second and third laws from the first law
(B) First and second laws from the third law
(C) Third and first laws from the second law
(D) All the three laws are independent of each other

3. When a body is stationary

(A) There is no force acting on it
(B) The force acting on it is not in contact with it
(C) The combination of forces acting on it balances each other
(D) The body is in vacuum

4. Two trains **A** and **B** are running in the same direction on parallel tracks such that **A** is faster than **B**. If packets of equal weights are exchanged between the two, then

(A) **A** will be retarded but **B** will be accelerated

(B) **A** will be accelerated but **B** will be retarded

(C) There will be no change in velocity of **A**, but **B** will be accelerated

(D) There will be no change in velocity of **B**, but **A** will be accelerated.

5. A body of mass 2 kg moving on a horizontal surface with initial velocity of 4 ms^{-1} comes to rest after two seconds. If one wants to keep this body moving on the same surface with a velocity of $4ms^{-1}$, the force required is

(A) 2 N (B) 4 N

(C) 0 N (D) 8 N

6. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0.15 kg., determine the impulse imparted to the ball. (Assume linear motion of the ball)

(A) 4.6 Ns (B) 3.6 Ns

(C) 2.6 Ns (D) 0

7. An object at rest in space suddenly explodes into three parts of same mass. The momentums of the two parts are $2p\hat{i} + p\hat{j}$. The momentum of the third part

(A) Will have a magnitude $p\sqrt{3}$. (B) Will have a magnitude $p\sqrt{5}$.

(C) Will have a magnitude p. (D) Will have a magnitude 2p.

8. A ball of mass m is thrown vertically upwards. What is the rate at which the momentum of the ball changes

(A) Zero (B) Mg

(C) Infinity (D) Data is not sufficient.

UNIT OF FORCE

Unit of force in SI system in Newton.

One Newton force is that force which when acted on a body of mass one kilogram produces on acceleration of $1m/sec^2$.

Unit of force in C.G.S system is dyne.

One dyne force is that force which when acted on a body of mass one gram produces an acceleration of $1cm/sec^2$

1 newton = 10^5 dynes

1 kg wt = g newton = 9.8N

Dimension of force is = $[M^1L^1T^{-2}]$

Gravitational unit of force - In SI system, kilogram weight (kg-wt) or kilogram force (kgf)

1 kgwt = 1 kg f = 9.8 newton.

In C.G.S system, gram-weight (gm - wt) or gram - force (gm f)

1 gmwt = 1gmf = 980 dyne

INERTIAL AND GRAVITATIONAL MASS

- Mass can be defined in two ways, by Newton's laws of motion and by Newton's law of gravitation
- According to Newton's second law of motion, if a constant force applied on a body produces an acceleration 'a' in that body then

  $\frac{F}{a}$ = mass (m)

- This mass is called **inertial mass**

  According to Newton's law of gravitational force is due to a body like earth the gravitational force of attraction F produces an acceleration g then. F = mg

  or $m = \frac{F}{g} = \frac{FR^2}{GM}$, $g = \left(\frac{GM}{R^2}\right)$, M = mass of earth, R = radius of earth.

- The mass is obtained from gravitational force F and acceleration due to gravity g is called **gravitational mass.**
- Even with the most sensitive instruments available no difference has been detected between the values of inertial and gravitational mass of a body i.e.

  Inertial mass = Gravitational mass.

General Expression for Force :

As $\vec{p} = m\vec{v}$

$$\Rightarrow \quad \vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + \frac{md\vec{v}}{dt}$$

Case - I :

When mass = constant, $\Rightarrow \frac{dm}{dt} = 0$

$$\vec{F} = \frac{d\vec{v}}{dt} = m\vec{a}$$

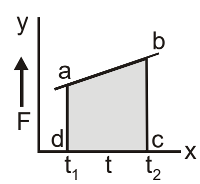
Case - II :

In variable mass system like Rocket propulsion, firing of bullets etc.

Relative speed is constant in these cases

$$\vec{F} = \vec{v}\frac{dm}{dt}$$

IMPULSE



- The impulse of constant force is equivalent to the product of the force and the time during which the force acts on the body.
- Impulse = (magnitude of force) time = Ft

- If the force change with time, then the impulse = $\int_0^t F dt$
- It is a vector quantity its unit is newton-sec.
- Relation between impulse and linear momentum

Impulse = $\int_0^t \vec{F} dt = \int_0^t \frac{d\vec{P}}{dt} dt = \int_0^t d\vec{P}$ = Change in momentum = $(m\vec{v} - m\vec{u})$

If a graph is drawn between force and time, then area between (F – t) curve and time axis is equal to the impulse.

$\therefore$ Impulse between t_1 and t_2 = area abcd.

SOLVED EXAMPLE

Example 1. A cricket ball of mass 150 gm. moves at a speed of 12 m/s and after getting lit by the bat it is deflected back at the speed of 20 m/sec. If the bat and the ball remained in contact for 0.02 sec then calculate the impulse and average force exerted on the ball by the bat.

[Assume the ball always moves normal to the bat].

Solution. According to the Example the change in momentum of the ball

$\Delta p = p_f - p_i = m(v - u) = 150 \times 10^{-3} [20 - (-12)] = 4.8$

$\therefore$ impulse = change in momentum = 4.8 N-s

and force = 240 N

Example 2. A ball of mass 150g moving with acceleration $20 m/s^2$ is hit by a force, which acts on it for 0.1 sec. The impulsive force is

(A) 0.5 N-s (B) 0.1 N-s

(C) 0.3 N-s (D) 1.2 N-s

Solution : **(C)**

Impulsive force = force × time = ma × t = 0.15 × 20 × 0.1= 0.3 N-s

Example 3. A force of 50 dynes is acted on a body of mass 5 g which is at rest for an interval of 3 seconds, then impulse is

(A) 0.15×10^{-3} N – s (B) 0.98×10^{-3} N – 3

(C) 1.5×10^{-3} N – s (D) 2.5×10^{-3} N – 3

Solution : **(C)**

Impulse = force × time = $50 \times 10^{-5} \times 3 = 1.5 \times 10^{-3}$ N – s

EXERCISE

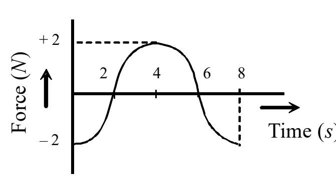
1. The force-time (F – t) curve of a particle executing linear motion is as shown in the figure. The momentum acquired by the particle in time interval from zero to 8 second will be

(A) – 2 π N–s

(B) + 4 π N–s

(C) 6 π N–s

(D) Zero



2. An object with a mass of 12.53 kg experiences a force of 16.41 N. What is the acceleration of the object?

(A) 1.30 m/s^2 (B) 37.58 m/s^2

(C) 4.13 m/s^2 (D) 0.76 m/s^2

3. An object with a force of 14.67 N on it is accelerating at 17.18 m/s^2. What is the mass of the object?

(A) 0.85 kg (B) 1.36 kg

(C) 0.34 kg (D) 252.03 kg

PSEUDO FORCE

- Those forces which do not actually act on the particles but appear to be acting on the particles due to accelerated motion of frame of reference, are called pseudo forces.
- Fictitious force = –(mass of a particle × acceleration of non-inertial frame of reference with respect to an inertial frame of reference)

Examples

- The additional force acting in rockets or lifts moving with accelerated motion is pseudo force.
- If a body is placed on a rotating frame of reference (the frame of reference on earth), the cariolis and centrifugal forces appear to be acting due to rotation of frame of reference. They are not real forces but appear to be acting due to rotation of frame of reference. Therefore they are pseudo forces.

THIRD LAW OF MOTION

To every action there is an equal and opposite reaction. Here by action and reaction, means force only, if an object exerts a force F on a second, then the second object exerts an equal but opposite force ion the first object.

Mathematically $\Rightarrow \vec{F}_{12} = -\vec{F}_{21}$

Here $\vec{F}_{12}$ is force acting on body 1 due to 2 and is $\vec{F}_{21}$ force acting on body 2 due to 1.

Significance of Newton's Laws:

(A) The first law tells us about the natural state of a body, which is motion along a straight line with constant speed or rest. It is also known as law of inertia.

(B) The second law tells us that if a body does not follow its natural state of motion then it is under the influence of other bodies, that is, a net unbalanced force must be acting on it.

(C) The third law tells us about the nature of force, that is, force exists in pair.

SOLVED EXAMPLE

Example 1. When a man jumps out of a boat, the boat is pushed away. Why?

Solution . This is due to Newton's third law of motion. When man jumps out of a boat, he presses / pushes/kicks the boat in the backward (opposite) direction and in turn, the reaction of the boat on the man, pushes him out of the boat.

Example 2. The force of action and reaction are equal and opposite but the motion of the body doesn't seize because

(A) Both action and reaction act on the same body in same direction.

(B) Both action and reaction act on the same body in opposite direction.

(C) Both action and reaction act on two different objects.

(D) None of these.

Solution. **(C)**

Action and reaction act on two different objects so that the point of action of the two forces is not same and hence law of vector addition can't be applied.

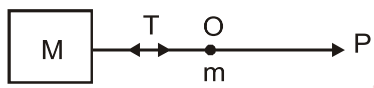
Example 3. A block of mass M is pulled along horizontal frictionless surface by a rope of mass m. Force P is applied at one end of rope. The force which the rope exerts on the block is

(A) $\frac{P}{M - m}$ (B) $\frac{MP}{(m + M)}$

(C) $\frac{P}{(m + M)}$ (D) $\frac{MP}{(M - m)}$

Soluton. (B)

The situation is shown in figure



Let a be the common acceleration of the system. Here

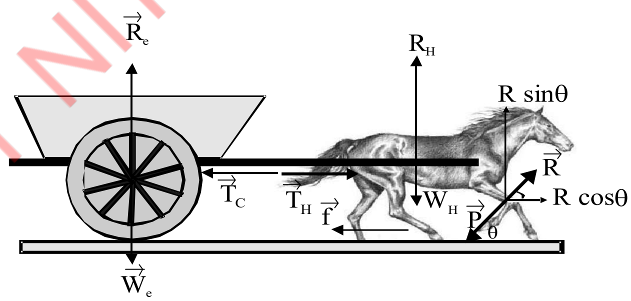
T = Ma for block, P – T = ma for rope

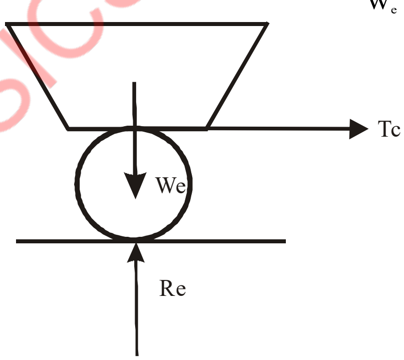
i.e. P – Ma or P = a (M + m) or a = $a = \frac{P}{(M + m)}$

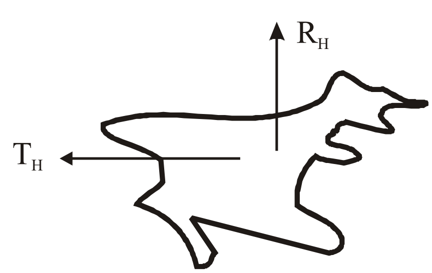
Now $T = \frac{MP}{(M + m)}$

APPLICATIONS OF NEWTON'S LAWS

1. **Horse and Cart Problem:** A horse pulls a cart. According to Newton's 3rd law of motion the cart pulls the horse with equal but opposite force. Then, how does the cart move?







$\overrightarrow{W_c}$ weight of cart & $\overrightarrow{R_c}$ normal reaction of cart.

$\overrightarrow{W_c} = \overrightarrow{R_c}$

$\overrightarrow{W_H} = \overrightarrow{R_H}$ ($W_H \rightarrow$ wt. of horse and $R_H \rightarrow$ normal reaction of horse)

$\vec{P}$ applied force & $\vec{R}$ is the reaction of $\vec{P}$ and $\vec{f}$ force of friction

Resolving R into two perpendicular components $R\sin\theta$ and $R\cos\theta$

We get, $R\sin\theta = mg$.

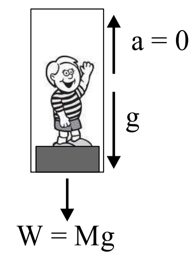
Now we are left with f and $R\cos\theta$ which are in opposite direction.

Cart will move only if $R\cos\theta > f$

MOTION IN A LIFT

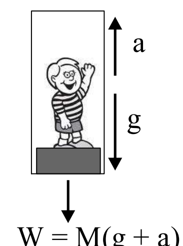
The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg is standing in a lift.

We consider the following cases :



Case (a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer O' in side the lift.

So apparent weight W' = Actual weight W.

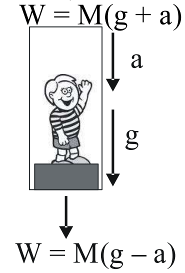


Case (b) If the lift accelerated (i.e. a = constant upwards acceleration) Then net force acting on the man are (i) weight W = Mg downward (ii) fictitious force F_0 = Ma downward

So apparent weight $W' = W + F_0$

or $W' = Mg + Ma = M(g + a)$

Effective gravitational acceleration $g' = g + a$

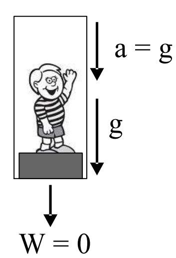


Case (c) If the lift accelerated downward with accelertion a < g : Then ficititious force F_0 = Ma acts upward while weight of mass W = Mg always acts downward, therefore

So apparent weight $W' = W + F_0$

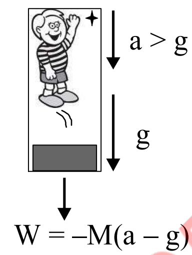
or $W' = Mg - Ma = M(g - a)$

Effective gravitational acceleration $g' = g - a$



Special case : If g = a then $W' = 0$ condition of weightlessness

Thus, in a freely falling lift the man will experience weightlessness.



Case (d) If lift accelerates downward with acceleration a > g

Then as in Case (c)

Apparent weight $W' = M(g - a) = -M(a - g)$ is negative, i.e, the man will be accelerated upward and his head will stick to the ceiling of the lift.

SOLVED EXAMPLE

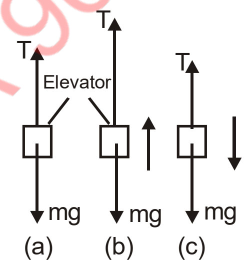
Example 1. The mass of an elevator (lift) is 500 kg. Calculate the tension in the cable of the elevator when the elevator is (i) stationary, (ii) ascending with an acceleration of 2.0 ms^{-2}, (iii) descending with the same acceleration. ($g = 9.8\ N\ kg^{-1}$)

Solution. The gravity - force on the elevator is $mg = 500 \times 9.8 = 4900$ N

When the elevator is stationary (acceleration is zero), the net force on it will be zero (fig.a) that is,

$T - mg = 0$

or $T = mg = 4900$ N



When the elevator is accelerated upward, the net force on it will be in the upward direction. Therefore, the tension T will be greater than the gravity force mg (fig.b) According to Newton's second law, the net force $(T - mg)$ will be equal to mass m × acceleration a. That is,

$T - mg = ma$

or $T = mg + ma = 4900 + 500 \times 2.0 = 5900$ N

When the elevator is accelerated downward, net force will be in the downward direction and the tension T will be less than mg (fig.c) Again by Newton's law, we have

$mg - T = ma$

or $T = mg - ma = 4900 - 500 \times 2.0 = 3900$ N

Example 2. An elevator and its load weigh a total of 1600 ℓb. Find the tension T in the supporting cable when the elevator, originally moving downward at 20 fts^{-1} is brought to rest with constant acceleration in a distance of 50 ft.

Solution. The elevator moving downward stops after a distance of 50 ft. Therefore, by the formula $v^2 = u^2 + 2as$, we have

$0^2 = 20^2 + 2a \times 50$

$\therefore\ a = -4ft\ s^{-2}$

Let T be the tension in the cable. Then, we have

$T = m\ (g - a)$

$mg - T = m\ (-a)$

$T = mg + ma$

Here m = 1600 ℓb. In FPS system, $g = 32\ ft\ s^{-2}$

$$\therefore\ T = 1600\ \{32 - (-4)\} = 57600 \text{ poundals} = \frac{57600}{32} = 1800\ \ell b.\ wt.$$

Example 3. A lift of mass 2000 kg is supported by thick steel ropes. If the maximum upward acceleration of the lift is 1.2 ms^{-2} and the breaking stress for the ropes is 2.8×10^8 N m^{-2}, what should be the minimum diameter of the ropes? (g = 9.8 ms^{-2})

Solution. When the lift is accelerated upward, the tension T in the rope is greater than the gravity-force mg. If the acceleration of the lift is a, then by Newton's law, we have

$T - mg = ma$

or $T = mg + mg = m(g + a)$

Mass of the lift m = 2000 kg and maximum acceleration a = 1.2 ms^{-2}. Hence maximum tension in the rope is

$T = 2000 (9.8 + 1.2) = 22,000$ N.

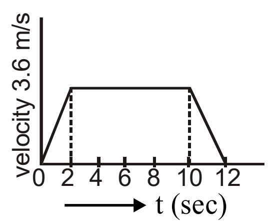
If r be the radius of the rope, then the stress will be $T/\pi r^2$

$$\therefore \frac{T}{\pi r^2} = 2.8 \times 10^8 \text{ Nm}^{-2} \text{ (given)}$$

$$\text{or } r^2 = \frac{T}{\pi \times (2.8 \times 10^8)} = \frac{22000}{(22/7) \times (2.8 \times 10^8)} = \frac{1}{4 \times 10^4}$$

$$\therefore r = \frac{1}{200} = 0.005 \text{ m.}$$

Example 4. A lift is going up, the total mass of the lift and the passenger is 1500 kg. The variation in speed of lift is as shown in fig. What will be the tension in the rope pulling the lift at (i) 1 sec, (ii) 6 sec, (iii) 11 sec.



Solution. A slope of v – t curve gives acceleration

$\therefore$ At, t = 1 sec

$$a = \frac{3.6 - 0}{2} = 1.8 \text{ m/s}^2$$

lift is moving up

$\therefore$ $T = m(g + a) = 1500 (9.8 + 1.8) = 17400$ N

at t = 6 sec a = 0

$\therefore$ $T = mg = 1500 \times 9.8 = 14700$ N

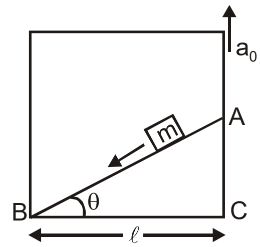
$$\text{at } t = 11 \text{ sec } a = \frac{0 - 3.6}{2} = -1.8 \text{ m/s}^2 \text{ lift is moving down.}$$

$\therefore$ $T = mg (g - a) = 1500 (9.8 - 1.8) = 12000$ N

Example 5. A lift is moving upwards with acceleration a_0. An inclined plane is placed in this lift. What is the time taken by a body of mass m in sliding down from the top of this plane to the bottom (If length of the base of the plane is '*1*' and angle is θ)

Solution. As the lift is moving up so the apparent weight of the body is m (g + a_0) and its component along the plane is m (g + a_0) sin θ due to which the acceleration down the plane is (g + a_0) sin θ

length of inclined plane AB = $\ell/\cos\theta$ using



$s = ut + \frac{1}{2}at^2$ and u = 0, a = (g + a_0) sin θ

$$\frac{\ell}{\cos\theta} = \frac{1}{2}(g + a_0)\sin\theta\, t^2$$

$$\therefore\ t = \sqrt{\frac{2\ell}{(g + a_0)\sin\theta\cos\theta}}$$

PULLEY

A single fixed pulley changes the direction of force only and in general assumed to be mass less and frictionless.

It is clear from example given below.

Example

A bucket of mass 25kg is raised by a 50 kg man in two different ways as shown in fig. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the bucket without the floor yielding?

Solution.

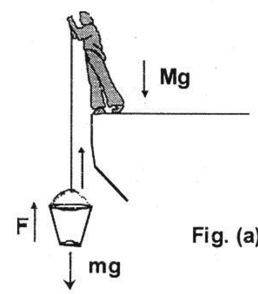


Fig. (a)

Here, mass of the bucket, m = 25 kg

mass of the man, M = 50 kg

Force applied to lift the bucket, F = mg = 25 × 9.8 = 245 N

Weight of the man, Mg = 50 × 9.8 = 490 N

fig (a) When the bucket is raised by the man by applying force F in upward direction, reaction equal and opposite to F will act on the floor in addition to the weight of the man.

Therefore, action on the floor = Mg + F = 490 + 245 = 735 N

Since the floor yields to a normal force of 700 N

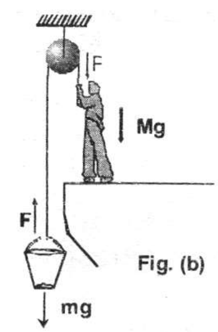


Fig. (b)

Fig. (b) When the bucket is raised by the man by applying force F over the rope (passed over the pulley) in downward direction, reaction equal and opposite to F will act on the floor. Therefore, action on the floor = Mg – F = 490 – 245 = 245 N

The mode (b) should be adopted by the man to lift the bucket.

Case I

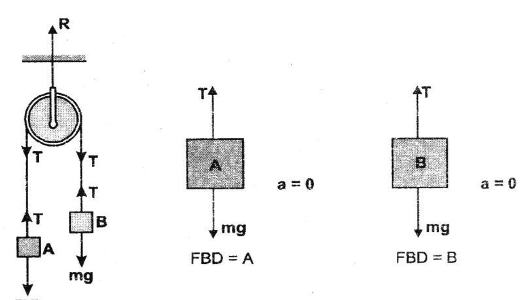
$m_1 = m_2 = m$

Tension in the string T = mg

Acceleration 'a' = zero

Reaction at the pulley

R = 2T = 2mg

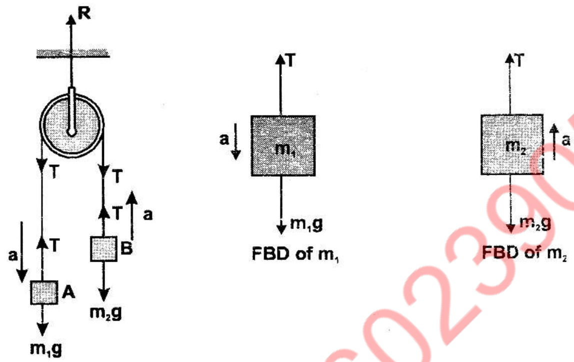


[Strength of support or suspension]

Case II

$m_1 > m_2$

for mass m_1, $m_1g - T = m_1a$ ....(A)



for mass m_2, $T - m_2g = m_2a$ ....(2)

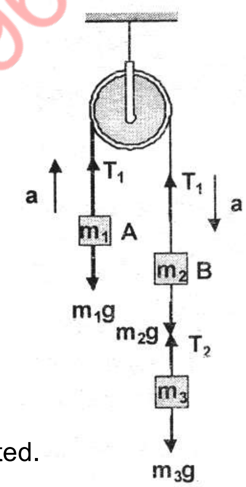
Solving, we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

or $T = \frac{(2m_1m_2)g}{(m_1 + m_2)}$

reaction at the pulley $R = 2T = \frac{4m_1m_2\,g}{(m_1 + m_2)}$

Case III



The forces acting on the system are shown in the fig.

For mass m_1 : $T_1 - m_1g = m_1a$ ...(1)

For mass m_2 : $m_2g + T_2 - T_1 = m_2a$ ...(2)

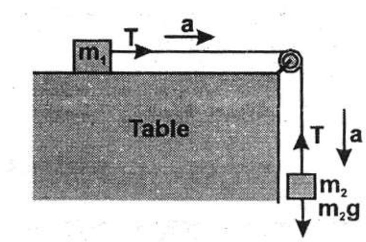
For mass m_3 : $m_3g - T_2 = m_3a$ ...(3)

Solving these equations, we get $a = \left(\frac{m_2 + m_3 - m_1}{m_1 + m_2 + m_3}\right) g$

Putting the value of 'a' in equation (i) & (iii) tension, T_1 & T_2 can be calculated.

Case IV

(a) Without friction :



For mass m_1 : $T = m_1a$

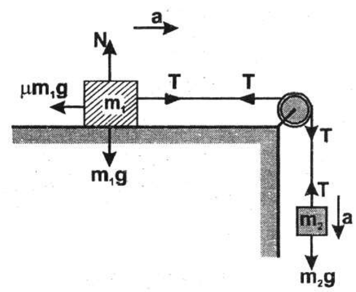
For mass m_2 : $m_2g - T = m_2a$

Solving, we get

Acceleration $a = \frac{m_2g}{(m_1 + m_2)}$

$$T = \frac{m_1m_2g}{(m_1 + m_2)}$$

(b) With friction :



(friction in between. surface and block)

For mass m_1 : $T - \mu m_1g = m_1a$

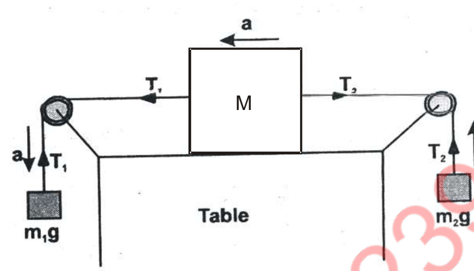
For mass m_2 : $m_2g - T = m_2a$

Solving we get

Acceleration $a = \frac{(m_2 - \mu m_1)\, g}{(m_1 + m_2)}$ or $T = \frac{m_1 m_2 (1+\mu)\, g}{(m_1 + m_2)}$

Case V

$(m_1 > m_2)$



$m_1g - T_1 = m_1a$ ...(i)

$T_2 - m_2g = m_2a$ ...(ii)

$T_1 - T_2 = Ma$ ...(iii)

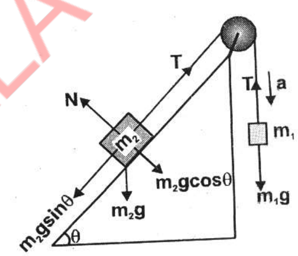
Solving (i), (ii) and (iii), we get

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2 + M)}$$

$$T_1 = m_1 g \left[\frac{2m_2 + M}{m_1 + m_2 + M}\right], \quad T_2 = m_2 g \left[\frac{2m_1 + M}{m_1 + m_2 + M}\right]$$

Case VI

Mass suspended over a pulley from another on an inclined plane.



(a) Without friction :

For m_1 : $m_1g - T = m_1a$

For m_2 : $T - m_2g \sin\theta = m_2a$

Solving, we get

Acceleration $a = \frac{(m_1 - m_2 \sin\theta)\, g}{(m_1 + m_2)}$

$T = \frac{m_1 m_2 (1 + \sin\theta) g}{(m_1 + m_2)}$

(b) With friction :

For m_1 : $m_1g - T = m_1a$

$N = m_2g \cos\theta$

$\mu N = \mu m_2 g \cos\theta$

For m_2 : $T - \mu m_2 g \cos\theta - m_2 g \sin\theta = m_2a$

Acceleration $a = \left[\frac{m_1 - m_2(\sin\theta + \mu\cos\theta)}{m_1 + m_2}\right] g$

Tension $T = \frac{m_1 m_2 [1 + \sin\theta + \mu\cos\theta] g}{(m_1 + m_2)}$

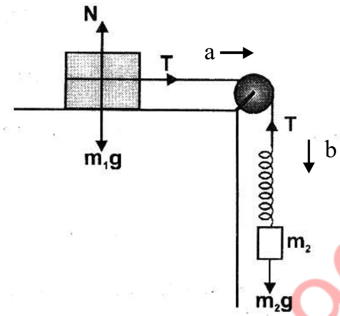
Case VII

Masses m_1 & m_2 are connected by a string passing over a pulley $(m_1 > m_2)$

$T - m_2g \sin\beta = m_2a$

Acceleration $a = \frac{g\,(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)}$

Tension $T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)\, g}{(m_1 + m_2)}$



Case VIII

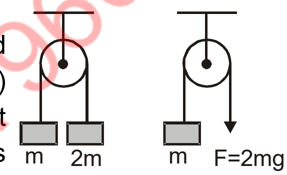
From case IV-(a) we know that the tension $T = \frac{m_1 m_2}{(m_1 + m_2)} g$

If x is the extension in the spring, then T = kx

$$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1 + m_2)}$$

SOLVED EXAMPLE

Example 1. The pulley arrangement of figures (a) and (b) are identical. The mass of the rope is negligible. In figure (a) the mass m is lifted by attaching a mass 2 m to the other end of the rope. In figure (b) mass m is lifted up by pulling the other end of the rope with a constant downward force F = 2mg. The ratio of acceleration in the two cases will be



(A) 1 : 1 (B) 1 : 3

(C) 3 : 1 (D) 1 : 2

Solution. Let in the case shown in figure (a), then tension in the string be T and acceleration produced is a, then

T – mg = ma ...(i)

and 2mg – T = 2ma ...(ii)

From equation (i) and (ii)

$a = \frac{g}{3}$

Let in the case shown in figure (b), the tension in the string be T' and acceleration be a', then

T' – mg = ma' ...(iii)

T' – 2mg = 0 ...(iv)

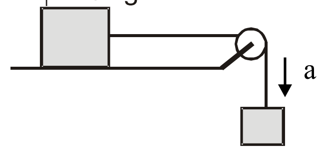
from equation (iii) and (iv)

a' = g

$\therefore \frac{a}{a'} = \frac{g/3}{g} = \frac{1}{3}$

$\therefore$ Answer is (B)

Example 2. A body of mass 10 kg is placed on the horizontal smooth table. A string is tied with it which passes over a frictionless pulley. The other end of the string is tied with a body of mass 5 kg. When the bodies move, the acceleration produced in them, is -



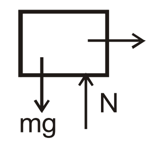
(A) 9.8 m/s^2 (B) 4.8 m/s^2

(C) 4.25 m/s^2 (D) 3.27 m/s^2

Solution. (i) $T = m_1 \times a$

$T = 10 \times a$

FBD (m_1)



(ii) $m_2g - T = m_2a$

$sg - 10a = sa$

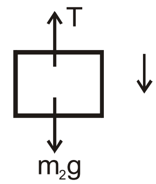
$sg = 15a$

$$a = \frac{5 \times 11}{15}$$

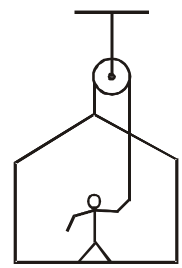
$a = 3.27$

$\therefore$ Answer is (D)

FBD



Example 3. The following figure shows a painter in a platform suspended along the building. When the painter pulls the rope the force exerted on the floor is 450 N while the weight of the painter is 1000N. If the weight of the platform is 250 N, the what will be the acceleration produced in the platform? ($g = 10\ m/s^2$)



(A) 4 m/s^2 (B) 2 m/s^2

(C) 5 m/s^2 (D) 6 m/s^2

Solution. Let the acceleration be a and mass of the painter $\frac{1000}{10}$ = 100 kg. If the pull applied to the rope by the painter is F, then the rope will also apply same amount of force. From Newton's law

$F + 450 - 1000 = 100a$

or $F - 550 = 100a$ ...(i)

Mass of platform = $\frac{250}{10}$ = 25 kg

$\therefore$ $F - 450 - 250 = 25a$

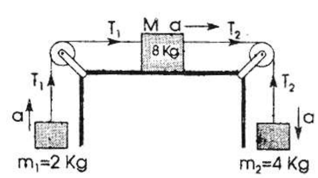
or $F - 700 = 25a$ ...(ii)

from equation (i) and (ii)

$a = 2 m/s^2$

$\therefore$ Answer is (B)

Example 4. In the following figure, if the table and pulley are frictionless and strings are weightless, and then the acceleration of the system will be-



(A) 3.2 m/s^2 (B) 2.5 m/s^2

(C) 1.8 m/s^2 (D) 1.4 m/s^2

Solution. From the figure,

$m_2g - T_2 = m_2a$ ...(i)

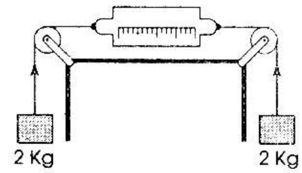
$T_1 - m_1g = m_1a$ ...(ii)

or $T_2 - T_1 = Ma$ ...(iii)

solving (i), (ii) and (iii)

$a = 1.4\ m/s^2$ ∴ Answer is (D)

Example 5. Two 2kg weights are suspended from a spring balance as shown in figure. The reading in the scale of the spring balance will be -



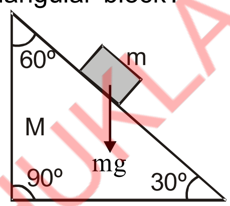
(A) zero (B) 4 kg-wt

(C) 2 kg-wt (D) 1 kg-wt

Solution. Spring balance is rest therefore tensions will be same in both strings. Let it be T. Since the weights are also at rest. So T – 2kg wt = 0. Here one weight, acts as a support of another weight.

∴ Answer is (C)

Example 6. A triangular block of mass M with angles 30º, 60º and 90º rests with its 30º – 90º side on a horizontal smooth table. A cubical block of mass m is placed on 30º – 60º side of the block. With what acceleration should M be moved relative to stationary table so that the mass m remains stationary relative to the triangular block?



(A) $5.66\ m/s^2$

(B) $4.32\ m/s^2$

(C) $9.8\ m/s^2$

(D) $4.9\ m/s^2$

Solution. Acceleration of the block of mass m along the inclined plane in downward direction

$= g \sin\theta = g \sin 30^\circ$

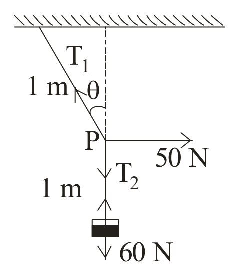
If the block m is to remain stationary, then

$g \sin 30^\circ = a \cos 30^\circ$

or $a = g\dfrac{\sin 30^\circ}{\cos 30^\circ} = g \tan 30^\circ$

$= 9.8 \times \dfrac{1}{\sqrt{3}} = \dfrac{9.8}{1.732} = 5.66\ m/s^2$ ∴ Answer is (A)

EXERCISE

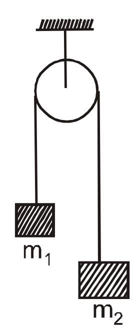


1. In the following fig. a mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction is applied at the midpoint P of the rope, as shown. What is the angle the rope will make with the vertical in equilibrium ? (Take g = 10 ms^{-2}). Neglect the mass of the rope.

(A) 40° (B) 50°

(C) 60° (D) 70°

2. Two masses 4.8 kg and 5 kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when left free to move ?



(A) 5 ms^2 (B) 9.8 ms^2

(C) 0.2 ms^2 (D) 4.8 ms^2

3. A body of mass m is suspended by two strings making angles α, β with the horizontal. Tensions in the two strings are

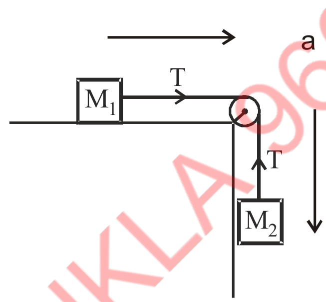
(A) $T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)} = T_2$

(B) $T_1 = \frac{mg\sin\beta}{\sin(\alpha+\beta)} = T_2$

(C) $T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}, T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$

(D) None of these

4. Two masses M_1 and M_2 are attached to a string which passes over a pulley attached to the edge of a horizontal table. The mass M_1 lies on the frictionless surface of the table. Let T be the tension in the string and 'a' the acceleration of the masses. Then which one of the following is correct.



(A) $a = \left[\frac{M_2}{M_1+M_2}\right]g$ and $T = \left[\frac{M_1M_2}{M_1+M_2}\right]g$

(B) $a = \left[\frac{M_1M_2}{M_1+M_2}\right]g$ and $T = \left[\frac{M_2}{M_1+M_2}\right]g$

(C) $a = \left[\frac{M_1+M_2}{M_1M_2}\right]g$ and $T = \left[\frac{M_1+M_2}{M_2}\right]g$

(D) $a = \left[\frac{M_1M_2}{M_1-M_2}\right]g$ and $T = \left[\frac{M_2}{M_1-M_2}\right]g$

5. Two masses m and 2m are attached with each other by a rope passing over a frictionless and mass less pulley. If the pulley is accelerated upwards with an acceleration 'a', what is the value of T?

(A) $\frac{g+a}{3}$

(B) $\frac{g-a}{3}$

(C) $\frac{4m(g+a)}{3}$

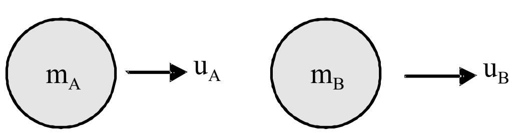
(D) $\frac{m(g-a)}{3}$

LAW OF CONSERVATION OF MOMENTUM

If no external force is acting on a body, then its linear momentum $\vec{p}$ remains conserved or constant i.e. if F = 0, then

$$\frac{d\vec{p}}{dt} = 0 \text{ or } \vec{p} = m\vec{v} = \text{Constant}$$

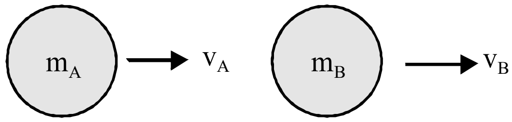
Let the initial velocity of body A is u_A and that of body B is u_B, after collision final velocity of A becomes v_A and that of B becomes V_B.



Then, Momentum of ball A before collision is m_Au_A

Momentum of ball B before collision is m_Bu_B

After collision ;



Momentum of ball A after collision is m_Av_A

Momentum of ball B after collision is m_Bv_B

The rate of change of momentum of ball A is $F_A = m_A \frac{(v_A - u_A)}{t}$

The rate of change of momentum of ball B is $F_B = m_B \frac{(v_B - u_B)}{t}$

According to third law, the force exerted by ball A on ball B is equal and opposite.

$F_A = -F_B$

$$m_A \frac{(v_A - u_A)}{t} = -m_B \frac{(v_B - u_B)}{t}$$

$$m_Av_A - m_Au_A = m_Bu_B - m_Bv_B$$

$$m_Au_A + m_Bu_B = m_Av_A + m_Bv_B$$

From the above equation it is clear that

Momentum before collision = Momentum after collision

If the two bodies stick to each other after collision then they will move with a common velocity v given by, $m_Au_A + m_Bu_B = (m_A + m_B)v$

$$v = \frac{(m_Au_A + m_Bu_B)}{(m_A + m_B)}$$

APPLICATIONS OF CONSERVATION OF MOMENTUM

1. A block of mass *m* is at rest in a gravity free space suddenly explodes into two parts in the ratio 1 : 3, if lighter particle has a speed of 9 m/s along +ve x-axis just after explosion, then the speed of heavier particle can be calculated as shown.

$$\vec{P}_{inital} = 0$$

$$\vec{P}_{final} = \left(\frac{m}{4}\right)9\,\hat{i} + \left(\frac{3m}{4}\right)\vec{v}$$

$$\vec{P}_{initial} = \vec{P}_{final}$$

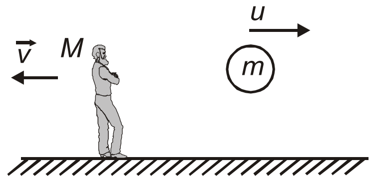
$$0 = \left(\frac{m}{4}\right)9\,\hat{i} + \left(\frac{3m}{4}\right)\vec{v}$$

$$\vec{v} = -3\hat{i}\ m/s$$

2. A man of mass *M* standing on a frictionless floor, throws a ball of mass *m* with a speed *u* along +ve x-axis as shown in the figure. The velocity of man, after he throws the ball is $\vec{v}$

$\vec{p}_{initial} = 0$

$\vec{p}_{final} = mu\hat{i} + M\vec{v}$

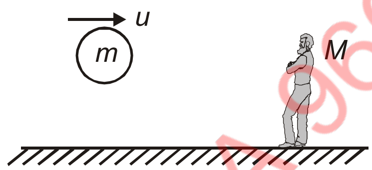


As $\vec{p}_{initial} = \vec{p}_{final}$

$$\vec{v} = \frac{-m\hat{i}}{M}$$

3. A ball of mass *m* moving with constant speed of *u* m/s (along +ve x-axis) is caught by man of mass *M* standing on a frictionless floor. The velocity acquired by the man is $\vec{v}$

$\vec{p}_{initial} = mu\hat{i}$



$\vec{p}_{final} = (m+M)\vec{v}$

$\vec{p}_{initial} = \vec{p}_{final}$

$$\vec{v} = \frac{mu\hat{i}}{(m+M)}$$

4. If a gun of mass *M* fires a bullet of mass *m* with speed *v*, then recoil speed of the gun is given by V = mv/M.

ROCKET PROPULSION

It is an example of variable mass-system. In a rocket, combustion chamber carries fuel and oxidising agent. When the fuel burns, a hot jet of gases emerges out forcefully from the small hole in the tail of the rocket.

Let *u* be the velocity of emerging gases relative to the rocket and (ΔM/Δt) be the rate of fuel consumption, then the upthrust on the rocket is F = u (ΔM/Δt).

Let M_0 be the initial mass (rocket + fuel), *M* and *v* be the mass and velocity of the rocket at any instant *t*, then $\vec{v} = -\vec{u}\log_e\left(\frac{M_0}{M}\right)$

RECOIL

Recoil is the 'kick' given by a gun when it is fired. In technical terms, this kick is caused by the gun's backward momentum, which exactly balances the forward momentum of the projectile. In most small arms, the momentum is transferred to the ground through the body of the shooter; while in heavier guns such as mounted machine guns or cannons, the momentum is transferred to the ground through a mounting system.

The change in momentum results in a force which, according to Newton's second law, is equal to the time derivative of the backward momentum of the gun. The backward momentum is equal to the mass of the gun multiplied by its reverse velocity. This backward momentum is equal, by the law of conservation of momentum, to the forward momentum of the eject of the gun.

RECOIL MOMENTUM AND RECOIL ENERGY

There are two conservation laws at work when a gun is fired: conservation of momentum and conservation of energy. Recoil is explained by the law of conservation of momentum, and so it is easier to discuss it separately from energy.

The recoil of a firearm, whether large or small, is a result of the law of conservation of momentum. Assuming that the firearm and projectile are both at rest before firing, then their total momentum is zero. Immediately after firing, conservation of momentum requires that the total momentum of the firearm and projectile is the same as before, namely zero. Stating this mathematically:

$p_f + p_p = 0$

where p_f is the momentum of the firearm and p_p is the momentum of the projectile. In other words, immediately after firing, the momentum of the firearm is equal and opposite to the momentum of the projectile.

Since momentum of a body is defined as its mass multiplied by its velocity, we can rewrite the above equation as:

$m_f \times v_f + m_p \times v_p = 0$

$v_f = -(m_p / m_f) v_p$

where:

m_f is the mass of the firearm

v_f is the velocity of the firearm immediately after firing

m_p is the mass of the projectile

v_p is the velocity of the projectile immediately after firing

A consideration of energy leads to a different equation. From Newton's second law, the energy of a moving body due to its motion can be stated mathematically as:

$E_t = 1/2\ m\ v^2$

where: m is the mass of the firearm system, or ejects and projectile after leaving the barrel v is its velocity

SOLVED EXAMPLE

Example 1. A bullet of mass 20 g is fired by a gun of mass 20 kg. If the muzzle speed of the bullet is 100 ms^{-1}, what is the recoil speed of gun?

Solution : Mass of gun, M = 20 kg,

Mass of bullet, m = 0.02 kg,

Speed of bullet, v = 100 ms^{-1}

Then recoil speed of gun is given by V

$$V = \frac{mv}{M} = \frac{0.02 \times 100}{20} = 0.1\ ms^{-1}$$

Example 2. A nucleus is at rest. All of a sudden it splits into two small nuclei. What is the angle at which these two nuclei fly apart?

Solution : Let M = mass of nucleus at rest

$\therefore$ Momentum of the nuclei before disintegration = M × 0 = 0

Let m_1 and m_2 be the mass of the two smaller nuclei and v_1 and v_2 be their velocities.

$\therefore$ Momentum of the nucleus after disintegration = $m_1v_1 + m_2v_2$.

According to the law of conservation of linear momentum $m_1v_1 + m_2v_2 = 0$

or $m_1v_1 = -m_2v_2$

The –ve sign shows that the velocities v_1 and v_2 must be of opposite sign i.e. the two products must be emitted in opposite direction. Thus, the angle between two nuclei is 180°.

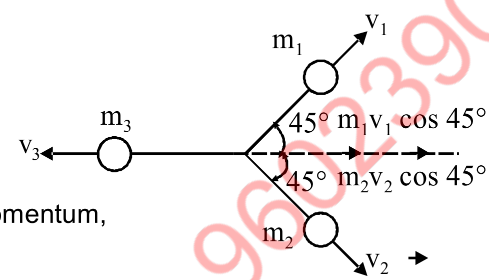
Example 3. A body of mass 1 kg initially at rest explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 ms^{-1} each. What is the velocity of heavier fragments?

Solution : Here $m_1 + m_2 + m_3 = 1kg$

Since $m_1 : m_2 : m_3 = 1 : 1 : 3$

$\therefore$ $m_1 = m_2 = 0.2$ kg, $m_3 = 0.6$ kg

$v_1 = v_2 = 30$ ms^{-1}, $v_3 = ?$



According to the law of conservation of linear momentum,

$$m_3v_3 = m_1v_1 \cos 45° + m_2v_2 \cos 45°$$

$$0.6v_3 = 0.2 \times 30 \times \frac{1}{\sqrt{2}} + 0.2 \times 30 \times \frac{1}{\sqrt{2}}$$

$\therefore$ $v_3 = 14.14$ ms^{-1}

Example 4. A body of mass 2kg moving with a speed of 100 m/s hits a wall and rebounds with the same speed. If the contact time is (1/50)s, the force applied on the wall is

(A) 10^4N (B) 2×10^4 N

(C) 4N (D) 8N

Solution. **(B)**

Initial momentum of the body = mv = 2 × 100 = 200 N-s

Final momentum = –200 N-s

Change of momentum of the body = –200 – (200) = – 400 m/s

Momentum given to the wall = +400 N-s

Time = $\frac{1}{50}$ s

$\therefore$ Momentum given per second

$\frac{400}{1/50} = 2 \times 10^4 \text{N} =$ Force applied on the wall

Example 5. The position x of a body of mass 2 kg varies with time t as

$x = (2t^2 + 3t - 4)$ m

Force acting on the body is -

(A) 4N (B) 8N

(C) 2N (D) 16 N

Solution. **(B)**

$x = (2t^2 + 3t - 4)$ m

Velocity $v = dx/dt = (4t + 3)$ m/s

Acceleration $a = d^2x/dt^2 = 4$ m/s^2

Force = mass × acceleration

= 2 × 4 = 8N

EXERCISE

1. A machine gun of mass 10 kg fires 20 g bullets at the rate of 10 bullets per second with a speed of 500 ms^{-1}. What force is required to hold the gun in position?

2. A bullet of mass 100 g is fired from a rifle of mass 20 kg with a speed of 50 ms^{-1}. Calculate the velocity of recoil of the rifle.

3. A bomb at rest explodes into three parts of the same mass The momentum of 2 parts are $-2p\hat{i}$ and $p\hat{j}$. Find the magnitude of the momentum of the third part.

4. When a ball is thrown upward, its momentum first increases and then decreases. In this cases law of conservation of momentum is valid or not.

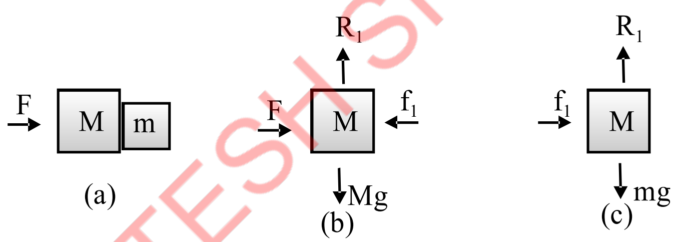
5. A hunter with a machine gun can fire 50 g bullets with a velocity of 800 ms^{-1}. A 40 kg tiger springs at him with a velocity of 10 ms^{-1}. How many bullets must he fire onto the tiger in order to stop it in his track?

FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free body diagram" F.B.D.

- In a specific Example, first we are required to choose a body and then we find the number of forces acting on it, and all the forces are drawn on the body, considering it as a point mass. The resulting diagram is known as free body diagram (FBD)

  For example, if two bodies of masses m and M are in contact and a force F on M is applied from the left fig. (A), the free body diagrams of M and m will be as shown in fig. (B) and (C)

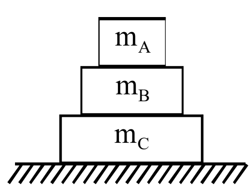


Important point :

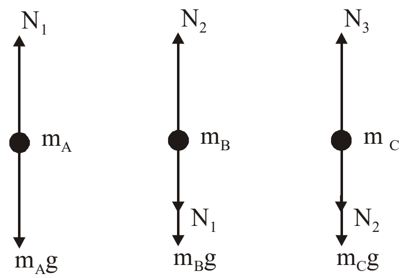
- Two forces in Newton's third law never occur in the same free-body diagram. This is because a free-body diagram shows forces acting on a single object, and the action-reaction pair in Newton's third law always acts on different objects.

SOLVED EXAMPLE

Example 1. Three blocks A, B and C having masses m_A, m_B and m_C are placed one over the other as shown in figure. Draw free body diagrams of all the three blocks.



Solution : Free body diagrams of A, B and C are shown below :

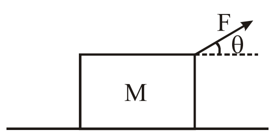


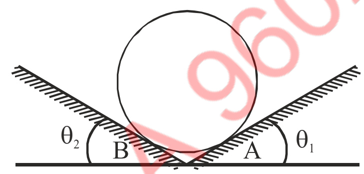
Here, N_1 = normal reaction between A and B.

N_2 = normal reaction between B and C.

N_3 = normal reaction between C and ground.

Example 2. Draw free body diagram situation shown in the figure.

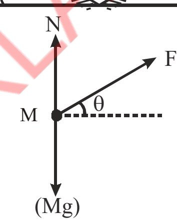
(i) 

(ii) 

Solution : (i) Forces acting on the block are

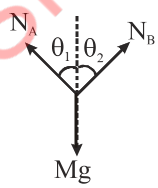
1. Applied force represented as F

2. Mg in vertically downward direction



Normal reaction of surface on block N in vertically upward direction

(ii) Let normal reaction at contact points A and B be N_A and N_B respectively. Mg in vertically downward direction



EQUILIBRIUM OF A PARTICLE

Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero. According to the first law, this means that, the particle is either at rest or in uniform motion or a system is said to be in translational equilibrium if it does not tend to undergo any further change of its own.

If two forces F_1 and F_2, act on a particle, equilibrium requires $F_1 = -F_2$

i.e. the two forces on the particle must be equal and opposite.

Equilibrium under three concurrent forces F_1, F_2, and F_3 requires that the vector sum of the three forces is zero or net force on body is zero $\sum \vec{F}_{net} = 0$

$F_1 + F_2 + F_3 = 0$. , i.e.

$$m\vec{a} = 0$$

$$m\frac{d\vec{v}}{dt} = 0$$

$\vec{v}$ = constant

i.e., if a body is in translational equilibrium it will be either at rest or in uniform motion in a straight line. If it is at rest, the equilibrium is called static. If it is in motion, the equilibrium is called dynamic.

Steps To Solve The Example of Connected Bodies

1. Make a simple sketch showing the body under consideration.
2. Identify the forces acting on the body, draw arrows on your sketch to show the direction of each force acting on the body.
3. Identify the direction of acceleration and resolve the forces along this direction and perpendicular to it
4. Find net force in the direction of acceleration and apply F = Ma to write equation of motion in that direction. In the direction of equilibrium take net force zero.
5. If needed write relation between accelerations of bodies given in the situation
6. Solve the written equations in 4 and 5 find unknown accelerations and forces.

CONSTRAINTS

Till now we had seen the case when accelerations of the different parts of a system are same. There are situations in which the accelerations of different parts of the system may not be the same. We get such situations in case of moveable pulleys or bodies in contact where each body is free to move. In such cases, a relationship between accelerations can be found by considering physical properties of system. We call such relations are constrained relation.

The procedure to determine constraint relation with this method is straightforward and given in the following steps :

Mathematical Procedure to Determine Constraint Relation:

Step I. Assume the direction of acceleration (or velocity) of each body (or particle),

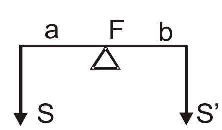
Step II. Locate the position of each particle from a fixed point e.g. centre of pulley,

Step III. Identify the constraint and write down the equation of constraint in terms of distance assumed.

CONDITION FOR BALANCE

Let S and S′ be the weights of scale pans and a and b be the arms of the balance respectively. As the beam remains horizontal when pans are empty, movements about fulcrum must be zero.

$\therefore$ S a = S′ b ....(i)



Suppose now that equal weights, W each, are put on the pans of the balance, the beam has to remain horizontal for balance to be true

$\therefore$ taking movements about F, we have

(S + W) a = (S′ + W) b ...(ii)

Using (i) or Wa = Wb or a = b i.e., arms of balance must be equal

Putting a = b, in equ, (i)

We have S = S′, weights of scale pans be equal

The method of Double weighing : The true weight of a body can be determined with the help of a false balance as follows :

(i) When arms are equal be weight of pans are unequal then the weight of body can be determined with the help of a false balance as follows :

Place the body having the true weight W in the left pan & counter poise it with standard weights = W_1,

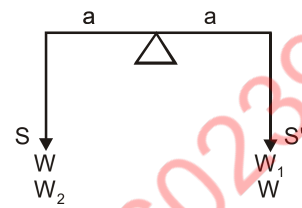
$\therefore$ $(S + W)\,a = (S' + W_1)a$ or $S + W = S' + W_1$

$S - S' = W_1 - W$ ...(i)

Now place the body in the right pan and counter poise it with standard weights = W_2

$\therefore$ $(S + W_2)\,a = (S' + W)\,a$ or $S + W_2 = S' + W$

$S - S' = W - W_2$ ...(ii)



equating R.H.S. of eq^n (i) and eq.(ii)

$$W = \frac{W_1 + W_2}{2}$$

$\therefore$ true weight of body is equal to the arithmetic mean of the two apparent weights.

(ii) When the beam remains horizontal but neither arms are equal nor weights of scale pans are equal

$\therefore$ $Sa = S'b$ ....(i)

Place the body with true weight W in the left arm and counter poise it with standard weights W_1

then $(S + W)a = (S' + W_1)b$

Using (i) we have $Wa = W_1b$ ....(ii)

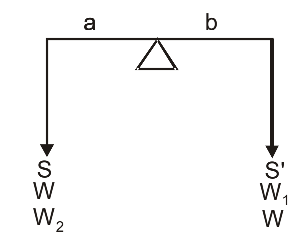
Now put the body in right pan and standard weights W_2 in the left arm to counter poise it.

$(S + W_2)\,a = (S' + W)\,b$

using (i) we have $W_2a = Wb$ or $Wb = W_2a$ ....(iii)

Multiply eq^n. (ii) and eq^n. (iii) vertically, we have

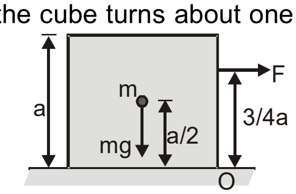
$W^2ab = W_1\,W_2\,ab$ or $W = \sqrt{W_1W_2}$



Thus the true weight of the body in this case is the geometric mean of the two apparent weights.

SOLVED EXAMPLE

Example 1. A cube having mass **m** and side **a** is placed on horizontal surface as shown in the figure. A horizontal force **F** is acting perpendicular to one of the surface, the force is at a point at a height of 3a/4 from the base. The minimum force required that the cube turns about one of its edges is



(A) (3/2) mg (B) mg

(C) (2/3) mg (D) 3/4 mg

Solution. The cube will turn about an edge O if the torque at O satisfies the following condition

$$F\,(3/4)\,a > mg\,a/2 \quad \text{or} \quad F > \frac{2}{3}\,mg$$

$\therefore$ F minimum = (2/3) mg

Hence the answer is (C)

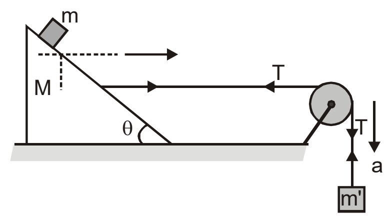
Example 2. What should be the value of m′ (mass of suspended block) so as to prevent the smaller block m from sliding over the triangular block of mass M. All surfaces are friction less and the string and pulley are light.

(A) $m' = \frac{m+M}{\cot\theta - 1}$

(2) $m' = \frac{\cot\theta - 1}{m+M}$

(3) $m' = \frac{m+M}{\tan\theta - 1}$

(4) $m' = \frac{M-m}{\cot\theta - 1}$



Solution. Writing force equations

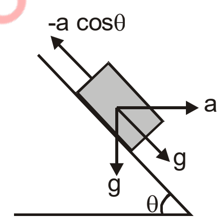
$m'a = m'g - T$ ...(i)

$T = (m + M)\,a$ ...(ii)

$m'a = m'g - (m + M)\,a$

$\therefore\ a\,(m' + m + M) = m'g$

or a = $a = \frac{m'g}{m'+m+M}$ ...(iii)



For the block having mass m, not to slide it is necessary that

$a\cos\theta = g\sin\theta$

$\frac{m'g}{m'+m+M}\cos\theta = g\sin\theta$

$\therefore\quad m' = (m' + m + M)\tan\theta$ or $m'(1 - \tan\theta) = (m + M)\tan\theta$

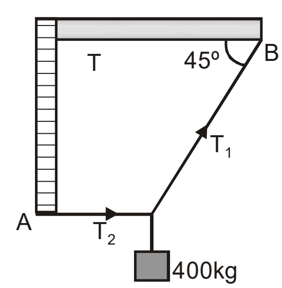
$m' = \frac{(m+M)\tan\theta}{1-\tan\theta} = \frac{m+M}{\cot\theta - 1}$ $\therefore$ Hence the answer is (A)

Example 3. A mass of 400 kg is suspended by two ropes from points A and B on the roof and the wall. The tension in rope OA is .

(A) 200 kg weight (B) 300 kg weight

(C) 400 kg weight (D) 600 kg weight

Solution. On resolving tension T_1, the horizontal and vertical components for T_1 are



$T_1 \sin 45° = Mg = 400 \times g$ and $T_1 \cos 45° = T_2$

$\tan 45° = \frac{400 \times g}{T_2} \Rightarrow T_2 = 400\,g = 400$ kg wt.

Hence the answer is (C)

Example 4. A rod AB whose length is 13 m rests on two perpendicular surfaces as shown in the figure and starts sliding. At a particular time end B is at a distance of 12 m from vertical surface and its velocity is 10 m/s. What is the velocity of A at this time :

(A) 10 m/s (upwards) (B) 24 m/s (upwards)

(C) 24 m/s (downwards) (D) 10 m/s (horizontal)

Solution. If length of rod is ℓ and at any time the coordinates of A and B are (0, y) and (x, 0) then $\ell^2 = x^2 + y^2$ now when Differentiating with respect to time $0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$$\therefore \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} \text{ or } \left(\frac{dy}{dt}\right) = V_A, \frac{dx}{dt} = V_B$$

$V_A = -\frac{x}{y}V_B$ or $V_B = 10$ m/s, x = 12 and y = 5

$V_A = -12/5 \times 10 = -24$ m/s negative (–) sign indicates that velocity V_A is downwards,

Hence the answer is (C)

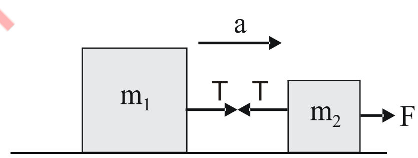
MOTION OF BODIES CONNECTED TOGETHER BY STRINGS

(a) For two bodies

Suppose two bodies of masses m_1 and m_2 are tied together and bodies are pulled by applying a force F on the body m_2.

If T is the tension produced in the string and 'a' is the acceleration produced in the system, then

acceleration = $\frac{\text{force}}{\text{total mass}} = \frac{F}{(m_1 + m_2)}$



Since acceleration is same for both bodies.

$\therefore$ For the body m_1,

$T = m_1 a$

For the body m_2

$F - T = m_2 a$

or $T = F - m_2 a = F - m_2 \frac{F}{(m_1 + m_2)}$

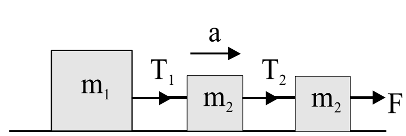
$\therefore$ Tension in the string

$$T = \frac{m_1 F}{(m_1 + m_2)}$$

(b) For three bodies

If three bodies of masses m_1, m_2 and m_3 are tied by strings and pulled by a force F, the acceleration produced in the system -

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$



For the body of mass m_1,

$T_1 = m_1 a$

For the body of mass m_2,

$T_2 - T_1 = m_2 a$

For the body of mass m_3

$F - T_2 = m_3 a$

Solving these equations

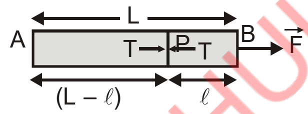
$$T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

and $$T_2 = \frac{(m_1 + m_2)\, F}{(m_1 + m_2 + m_3)} = (m_1 + m_2)\, a$$

Rope lying on a horizontal surface

(a) A uniform rope of length L which is lying on a frictionless table is pulled by applying a constant force

Let the mass of the rope be M and its length be L. So the mass per unit length of the rope is (M/L)



If T is the tension in the rope at a distance ℓ from end B and acceleration of the rope is a, then for part PB

$$F - T = \text{mass of part PB} \times a = \frac{M}{L}\ell a$$

For the part AP

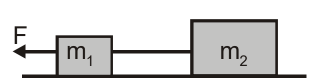
$$T = \text{mass of part AP} \times a = \frac{M}{L}(L - \ell)a$$

$$T = \frac{(L-\ell)}{\ell}(F - T)$$

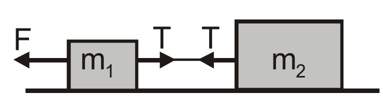
or $$T = \left(1 - \frac{l}{L}\right) F$$

SOLVED EXAMPLE

Example 1. Two bodies whose masses are m_1 = 50 kg and m_2 = 150 kg are tied by a light string and are placed on a friction less horizontal surface. When m_1 is pulled by a force F, an acceleration of 5 ms^{-2} is produced in both the bodies. Calculate the value of F. What is the tension in the string?



Solution. The force F is pulling both the bodies together. Hence if the acceleration produced in the direction of force be a, then by Newton's law of motion, we have



(net) force = mass × acceleration

$F = (m_1 + m_2) \times a \; = (50 + 150) \times = 1000$ N

To determine the tension in the string, we have to consider the force acting on the bodies separately. When $\mathbf{m_1}$ is pulled by the force **F**, then $\mathbf{m_1}$ pulls $\mathbf{m_2}$ through the string by a force **T**. This force is the tension in the string which acts on $\mathbf{m_2}$ in the forward direction (see fig) $\mathbf{m_2}$ also pulls $\mathbf{m_1}$ by the same (reactionary) force **T**. Hence the tension **T** of the string acts on $\mathbf{m_1}$ in the backward direction.

Thus, a 'net' force **F – T** acts on $\mathbf{m_1}$ in the forward direction. Hence, by Newton's law applied for $\mathbf{m_1}$ alone, we have

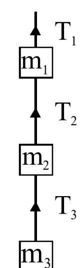
$F - T = m_1 a$

or $T = F - m_1 a = 1000 - (50 \times 5) = \mathbf{750\ N}$

We can determine **T** also by applying Newton's law for the motion of $\mathbf{m_2}$ alone. On $\mathbf{m_2}$ there is a net force **T** in the forward direction

$\therefore\ T = m_2 a = 150 \times 5 = 750$ N

Example 2. The masses m_1, m_2 and m_3 of the three bodies shown in fig. are 5, 2, 3 kg respectively. Calculate the values of the tension T_1, T_2 and T_3 when (a) the whole system is going upward with an accelerating of $2\,ms^{-2}$, (b) the whole system is stationary ($g = 9.8\ ms^{-2}$)



Solution. (a) All the three bodies m_1, m_2 and m_3 are moving upward together. The force pulling the system upward is T_1 and the downward gravity-force is $(m_1 + m_2 + m_3)$ g. Hence the net force on the system is $T_1 - (m_1 + m_2 + m_3)$ g. According to Newton's second law, this force is equal to total mass × acceleration. If a be the acceleration of the system in the upward direction, then

$T_1 = (m_1 + m_2 + m_3)\, g = (m_1 + m_2 + m_3)\, a$

or $T_1 - (5 + 2 + 3) \times 9.8 = (5 + 2 + 3) \times 2$

$\therefore\ T_1 = 20 + 98 = 118$ N

The force pulling m_2 and m_3 in the upward direction is T_2 and the gravity-force of them is $(m_2 + m_3)$ g. Hence the net force in the upward direction is $T_2 - (m_2 + m_3)$ g . Again, by Newton's law, we have

$T_2 - (m_2 + m_3)\, g = (m_2 + m_3) g$

$T_2 - (2 + 3) \times 9.8 = (2 + 3) \times 2$

$T_2 = 10 + 49 = 59$ N

The net force on m_3 in the upward direction is $T_3 - m_3 g$.

Hence by Newton's law, we have

$T_3 - m_3 g = m_3 a$

or $T_3 - 3 \times 9.8 = 3 \times 2$

$\therefore\ T_3 = 6.0 + 29.4 = 35.4$ N

(b) If the whole system is stationary (or moving with uniform velocity), then a = 0. Hence from equation. (i), (ii) and (iii), we have

$T_1 = (m_1 + m_2 + m_3)\, g = 10 \times 9.8 = 98$ N

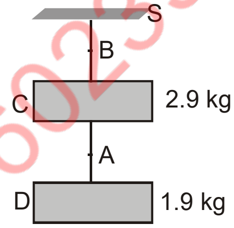
$T_2 = (m_2 + m_3)\, g = 5 \times 9.8 = 49$ N

$T_3 = m_3\, g = 3 \times 9.8 = 27.4$ N

Example 3. Two blocks of masses 2.9 kg and 1.9 kg are suspended from a rigid support **S** by two inextensible wires each of length 1 m. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg m^{-1}. The whole system of blocks, wires and support have an upward acceleration of 0.2 ms^{-2}. Acceleration due to gravity is 9.8 ms^{-2}.

(i) Find the tension at the mid-point of the lower wire.

(ii) Find the tension at the mid-point of the upper wire.



Solution. (i) Suppose, the tension at the point A is T_A. Then

$T_A - mg = ma$

or $T_A = m\,(a + g)$

where m = 1.9 kg + mass of the wire of length

$AD = 1.9 + \frac{1}{2} \times 0.2 = 2.0$ kg

$\therefore\ T_A = 2.0 \times (0.2 + 9.8\ ) = 20$ N

(ii) Suppose, the tension at the point B is T_B, then

$T_B = M\,(a + g)$

where M = 2.9 kg + 1.9 kg + mass of the wire CD = 2.9 + 1.9 + 0.2 = 5.0 kg

$\therefore\ T_B = 5.0 \times (0.2 + 9.8) = 50$ N

EXERCISE

1. A body is in equilibrium under the action of three forces on it. For this it is necessary that the three forces

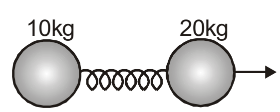
(A) are in or should be in a straight line

(B) should pass through the same point and do not lie in the same plane

(C) should pass through the same point and lie in the same plane

(D) should be mutually perpendicular to each other and pass through a common point

2. Two masses 10 kg and 20 kg are connected with a mass less spring as shown in the figure. A force of 200 N is acting on the mass 20 kg. When the acceleration of 10 kg mass is 12m/s^2, the acceleration of 20 kg mass is -



(A) 12 m/sec^2 (B) 4 m/sec^2

(C) 10 m/sec^2 (D) zero

3. A lift is descending with an acceleration 'a'. A person standing in it drops a book. The acceleration of the book relative to floor of the lift will be (take acceleration due to gravity = g)

(A) g (B) a

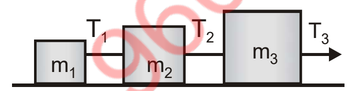
(C) g – a (D) g + a

4. Ratio of weight of a man in a stationary lift & weight of a man in a lift moving down-ward with an acceleration a is 3 : 2 then acceleration of lift is -

(A) g/3 (B) g/2

(C) g (D) 2g

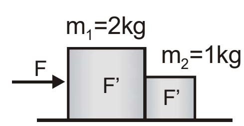
5. Three block are connected as shown in fig., on a horizontal frictionless table and pulled to the right with a force T_3 = 60 N. If m_1 = 10 kg. m_2 = 20 kg. m_3 = 30 kg. the tension T_2 is -



(A) 10 N (B) 20 N

(C) 30 N (D) 60 N

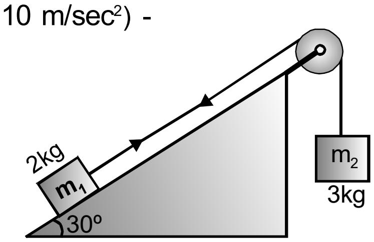
6. In the following figure, two blocks on m_1 = 2kg and m_2 = 1kg are in contact with a frictionless table. A horizontal force F = 3N is applied to mass m_1, the contact force between m_1 and m_2 will be -



(A) 1N (B) 2N

(C) 3N (D) zero

7. A block of mass m_1 = 2kg on a smooth inclined plane at angle 30° is connected to a second block of mass m_2 = 3kg by a cord passing over a frictionless pulley as shown in figure. The acceleration of each block is (assume g = 10 m/sec²) -



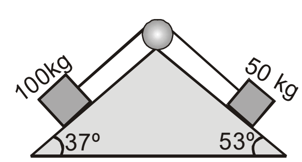
(A) 2 m/sec²

(B) 4 m/sec²

(C) 6 m/sec²

(D) 8 m/sec²

8. Two blocks are connected by a cord passing over a small frictionless pulley and resting on frictionless planes as shown in the figure. The acceleration of the blocks is -



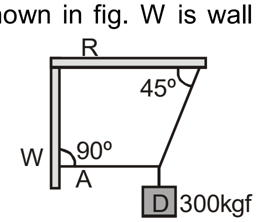
(A) 0.33 m/s²

(B) 0.66 m/s²

(C) 1 m/s²

(D) 1.32 m/s²

9. A block D of weight 300 kg is hanged with strings A & B as shown in fig. W is wall and R is rigid support. Tension in string A is -



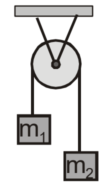
(A) zero (B) 150 kg

(C) 300 kg (D) 400 kg

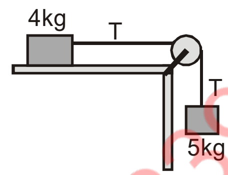
10. Two masses are suspended vertically on a pulley their acceleration is -

(A) $\frac{m_1}{m_2}g$ (B) $\frac{m_2}{m_1}g$

(C) $\left(\frac{m_2-m_1}{m_1+m_2}\right)g$ (D) $\left(\frac{m_1+m_2}{m_2-m_1}\right)g$



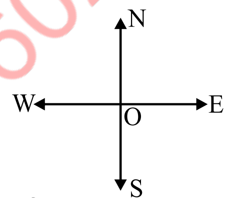
11. Two bodies of 5kg and 4kg are tied to a string as shown in fig. If the table and pulley both are smooth, acceleration of 5kg body will be equal to -



(A) g (B) g/4

(C) 4g/9 (D) 5g/9

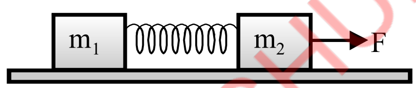
12. A body of mass 2kg has an initial velocity of 3 m/s along OE and it is subjected to a force of 4 newton in a direction perpendicular to OE. The distance of body from O after 4 seconds will be -



(A) 12 metres (B) 20 metres

(C) 8 metres (D) 48 metres

13. Two blocks of mass m_1 & m_2 connected with a light spring, placed on a frictionless table. They are left after pulling, ratio of acceleration produced in them -



(A) $\frac{m_1}{m_2}$ (B) $\frac{m_2}{m_1}$

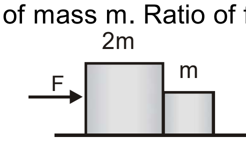
(C) $\frac{m_1-m_2}{m_1+m_2}$ (D) $\frac{4m_1m_2}{(m_1+m_2)^2}$

14. A rope of length L is pulled by a constant force F. What is the tension in the rope at a distance x from the end where the force is applied -

(A) $\frac{Fx}{L-x}$ (B) $\frac{FL}{L-x}$

(C) $\frac{FL}{x}$ (D) $\frac{F(L-x)}{x}$

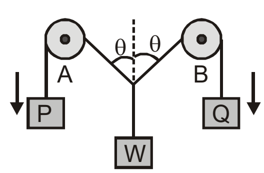
15. Two blocks at mass m & 2m are kept on a smooth horizontal plane. First force F is applied on block of mass 2m secondly same force F is applied on block of mass m. Ratio of forces between blocks in above two conditions.-



(A) 1 : 1 (B) 1 : 2

(C) 1 : 3 (D) 1 : 4

16. A weight W is tied to two strings passing over the frictionless pulleys A and B as shown in the figure. If weights P and Q move downwards with speed V, the weight W at any instant rises with the speed -



(A) V cosθ

(B) 2V cosθ

(C) V/cosθ

(D) 2V/cosθ

COMMON FORCES IN MECHANICS

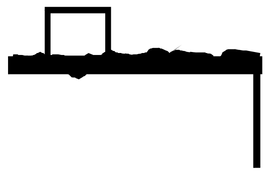
In mechanics, we encounter several kinds of forces. Gravitational force - It is the force of attraction between any two bodies in the universe. This force can act at a distance without the need of any intervening medium. All other forces common in mechanics are contact forces which satisfy Newton's 3rd law, like - Friction Force.

FRICTION

It is that opposing force which comes into play when one body tries or actually moves over the surface of another body.

What Causes Friction?

The surfaces of bodies are never perfectly smooth. Even a very smooth surface seen under a microscope, is found to have depressions and projections as shown in the figure below.



Enlarged view of apparently smooth surfaces in contact- The interlocking of the irregularities of the surfaces in contact causes friction.

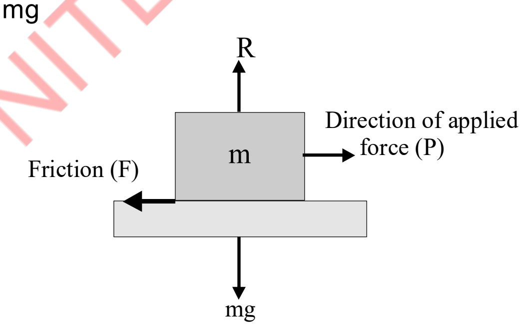
STATIC FRICTION

The opposing force that comes into play when one body tends to move over the surface of another, but the actual motion has yet not started is called static friction.

- In this case frictional force (F) is equal an opposite to the applied force (P).
- So as long as F = P body doesn't move.

$F \propto R$

Where F = P & R = mg



The static friction between two contact surfaces is given by $f_s < \mu_s N$, where N is the normal force between the contact surfaces and μ_s is a constant which depends on the nature of the surfaces and is called **'coefficient of limiting friction'**.

Static Friction - A Self-adjusting Force

Consider a block of wood lying on a plane horizontal table. There is no force of friction as the block of wood is stationary. Push the block lightly with the hand. The block does not move because the force applied by the hand is exactly balanced by the force of friction acting horizontally in the opposite direction. Now push the block a little harder, still the block does not move as the force of friction increases so as to become equal and opposite to the applied force.

Static friction acts on stationary objects. Its values satisfy the condition

$f_s < \mu_s N$

μ_s – Co-efficient of limiting friction

Static friction takes its peak value ($f_{s(max)} = \mu_s N$) when one surface is *'about to slide'* on the other. Static friction in this case is called limiting friction.

- **LIMITING:** Limiting friction is the maximum opposing force that comes into play, when one body is just at the verge of moving over the surface of another body (actual movement is not there).
- The maximum value of static friction, when motion is impending, is sometimes referred to as **limiting friction.**

Laws of Limiting Friction

1. The magnitude of the force of limiting friction (F) between any two bodies in contact is directly proportional to the normal reaction (R) between them.
2. The direction of force of limiting friction is always opposite to the direction of in which one body is at the verge of moving over the other.

   The force of limiting friction is independent of the apparent area of contact, so long as normal reaction between the two bodies in contact remains the same. The force of limiting friction between the two bodies in contact depends on the nature of the material of the surfaces in contact and their roughness or smoothness.

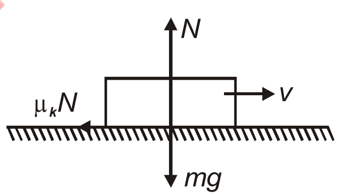
KINETIC FRICTION (μ_K)

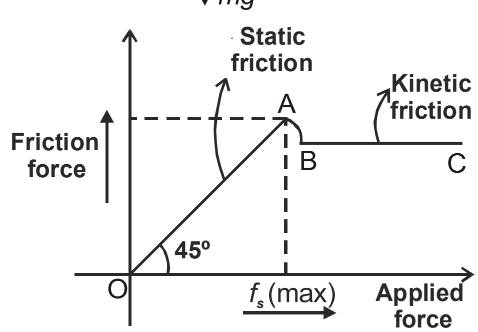
It is that opposing force which comes into play when one body actually moves over the surface of another body.

- **SLIDING:** It is that opposing force which comes into play when one body actually slides over the surface of another body.
- **ROLLING:** It is that opposing force which comes into play when one body actually rolls over the surface of another body.

  $f_k = \mu_k N$

The relative motion of a contact surface with respect to each other is opposed by a force given by $f_k = \mu_k$ N, where N is the normal force between the contact surfaces and μ_k is a constant called **'coefficient of kinetic friction'**, which depends, largely, on the nature of the contact surfaces.



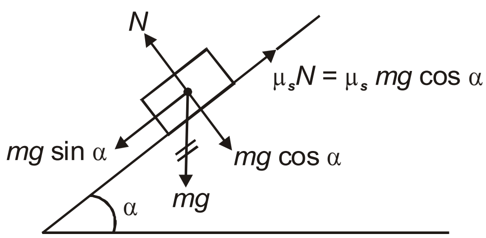


Graphical representation of variation of friction force with the force applied on a body

OA = Static Friction
AB = Limiting Friction
BC = Kinetic Fricition

ANGLE OF REPOSE

Consider a situation in which a block is placed on an inclined plane with co-efficient of friction 'μ' then the maximum value of angle of inclined plane for which the block can remain at rest is defined as angle of repose.



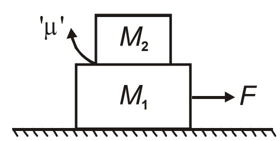
Two blocks placed one above the other

Case - I :

(a) $F \le \mu(M_1 + M_2)g$

Both blocks move together with same acceleration

$$a = \frac{F}{M_1 + M_2}$$



Smooth

$a_{max} = \mu g$

(b) $F > \mu(M_1 + M_2)g$

M_2 moves with constant acceleration $a_2 = \mu g$

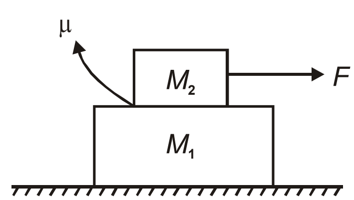
M_1 moves with acceleration $a_1 = \dfrac{F - \mu M_2 g}{M_1}$

M_2 slips backward on M_1.

Case - II :

(a) $F \le \dfrac{\mu(M_1 + M_2)M_2}{M_1} g$, both blocks move together with acceleration $a = \dfrac{F}{M_1 + M_2}$ with

$a_{max} = \dfrac{\mu M_2}{M_1} g$.



Smooth

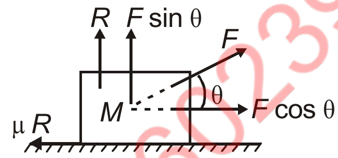
(b) $F > \dfrac{\mu(M_1 + M_2)M_2 g}{M_1}$,

M_1 moves with constant acceleration $a_1 = \frac{\mu M_2}{M_1} g$

M_2 moves with acceleration $a_2 = \frac{F - \mu M_2 g}{M_2}$

M_2 slips forward on M_1.

Minimum force required to move a body on a rough horizontal surface

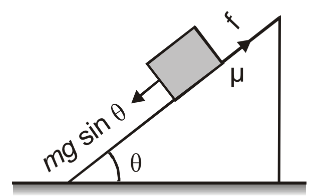


$F \cos\theta > \mu R$

$$F \geq \frac{\mu mg}{\cos\theta + \mu \sin\theta}$$

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} \text{ at } \theta = \tan^{-1}(\mu)$$

BLOCK ON INCLINED PLANE

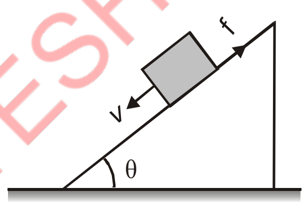


1. If $\mu \geq \tan\theta$, the block will remain stationary on the inclined plane; and the frictional force acting on the block will be equal to $mg \sin\theta$ and static in nature

   $f = mg \sin\theta$

2. If a block slides down an inclined plane with constant velocity, the frictional force acting on the block is kinetic in nature and is equal to $mg \sin\theta$

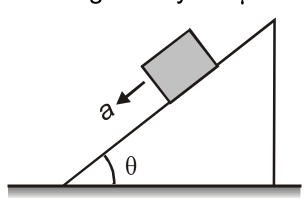
   $\Rightarrow f = mg \sin\theta$



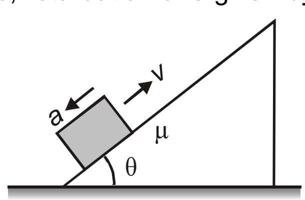
3. If $\mu < \tan\theta$, the block will slide down the plane with acceleration a equal to

   $a = g \sin\theta - \mu g \cos\theta$

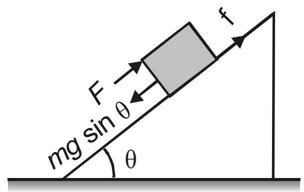
   Frictional force is kinetic in nature and is given by $f = \mu N = \mu mg \cos\theta$ (less than $mg \sin\theta$)



4. If a block is projected up the plane, retardation a is given by $a = g \sin\theta + \mu g \cos\theta$.

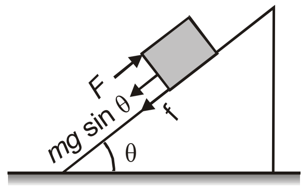


5. If *mg* sin θ exceeds frictional force, the block tends to slide down. The minimum force required to prevent sliding is F_{min} = mg sin θ – μ mg cos θ



6. If you try to push a block up the plane, frictional force and *mg* sinθ both opposes the upward motion of the block, hence, the minimum force required to move up is given by

F_{min} = mg sin θ + μ mg cos θ



Note : In case 5-6, if F lies between mg sin θ = μ mg cos θ and mg sin θ + μ mg cos θ, the block remains stationary on the inclined plane.

Friction is a Necessity

1. Walking will not be possible without friction. Our foot pressing the ground will only slip.
2. No two bodies will stick to each other without friction.
3. Brakes of the vehicles will not work without friction.
4. Nuts and bolts for holding the parts of machinery together will not work.
5. Writing on black board or on paper will also not be possible without friction.
6. The transfer of motion from one part of a machine to the other part through belts will not be possible without friction.
   A. Adhesives will lose their purpose.
   B. Cleaning with sand paper will not be possible without friction.

FRICTION IS AN EVIL

1. Friction always opposes the relative motion between any two bodies in contact. Therefore, extra energy has to be spent to overcome friction. Thus friction involves unnecessary expense of energy. It means output is always less than the input.
2. Friction causes wear and tear of parts of machinery in contact. Thus their life time reduces.
3. Frictional forces result in the production of heat, which causes damage to the machinery.

METHODS OF CHANGING FRICTION

(i) By polishing

(ii) By lubrication

(iii) By proper selection of materials

(iv) By streamlining

(v) By using ball bearings

Some of the ways of increasing friction:

1. On a rainy day, we throw some sand on the slippery ground. This increases friction between our feet and the ground. The chance of slipping gets reduced.
2. Similarly, sand is spread on tracks covered with snow. Forces of friction between the wheels and the track increases and driving become safer.
3. In the manufacture of tyres, synthetic rubber is preferred because its coefficient of friction with the road is larger. Proper threading of the tyres also increases the force of friction between the tyres and the road.

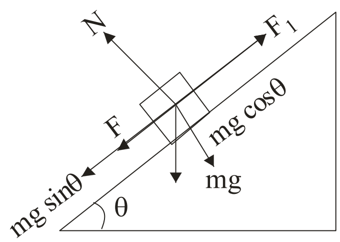
SOLVED EXAMPLE

Example 1. The force required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is . The inclination of the plane is

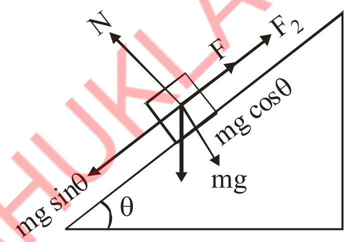
(A) $\tan^{-1}(\mu)$ (B) $\tan^{-1}(\mu/2)$

(C) $\tan^{-1}(2\mu)$ (D) $\tan^{-1}(3\mu)$

Solution. **(D)**



Case (a)



Case (b)

mg sin θ = $F_1 - \mu N$

N = mg cos

mg sin + mg cos = F_1

In second case (b)

N + F_2 = mg sin

mg cos + F_2 = mg sin

or F_2 = mg sin $-\mu mg\cos\theta$

but $F_1 = 2F_2$

therefore

$$mg\sin\theta + \mu mg\cos\theta = 2(mg\sin\theta - \mu mg\cos\theta)$$

$$mg\sin\theta = 3\mu mg\cos\theta$$

$$\text{or } \tan\theta = 3\mu$$

$$\theta = \tan^{-1}(3\mu)$$

Example 2. Automobile tyres have generally irregular projections over their surfaces. Explain why?

Solution . This is done to increase the force of friction. If the tyres do not have irregular projections, then the force of friction between the tyres and the road will be very small/less. Due to the small friction, the automobile may skid off the road when brakes are applied.

Example 3. Is large brake on a bicycle wheel more effective than a small one? Explain.

Solution. The force of friction is independent of the area of contact. Thus large brake and the small brake will have the same effect. However, the small brake may go out of order earlier, because of the faster wear and tear.

EXERCISE

1. A uniform chain of length L lies on a table. If the coefficient of friction is μ, then the maximum length of the chain which can hang from the edge of the table without the chain sliding down is -

(A) $\frac{\mu L}{\mu - 1}$ (B) $\frac{\mu L}{\mu + 1}$

(C) $\frac{L}{\mu - 1}$ (D) $\frac{L}{\mu}$

2. When a body slides down an inclined plane with coefficient of friction as μ, then its acceleration is given by -

(A) $g(\mu \sin\theta + \cos\theta)$ (B) $g(\mu \sin\theta - \cos\theta)$

(C) $g(\mu \sin\theta + \mu\cos\theta)$ (D) $g(\sin\theta - \mu\cos\theta)$

3. Work done in moving a body up an inclined rough plane (μ) of length S will be -

(A) $mg(\sin\theta + \mu\cos\theta)S$ (B) $mg(\mu\sin\theta + \cos\theta)S$

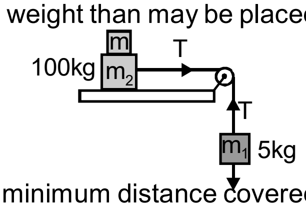
(C) $mg(\mu\sin\theta - \cos\theta)S$ (D) $mg(\sin\theta - \mu\cos\theta)S$

4. An object takes n times more time to slide down a 45º inclined rough surface as it takes to slide down a perfectly smooth 45º inclined surface. The coefficient of kinetic friction between the object and the incline is given by

(A) $\sqrt{(1 - n^2)}$ (B) $\sqrt{\left(1 - \frac{1}{n^2}\right)}$

(C) $(1 - n^2)$ (D) $\left(1 - \frac{1}{n^2}\right)$

5. Two masses m_2 = 10kg and m_1 = 5kg are connected by a string passing over a pulley as shown. If the coefficient of friction be 0.15, then the minimum weight than may be placed on m_2 to stop motion is -



(A) 18.7 kg (B) 23.3 kg

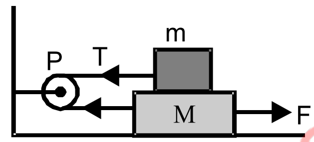
(C) 32.5 kg (D) 44.3 kg

6. The coefficient of friction between the tyres and road is 0.4. The minimum distance covered before attaining a speed of 8 ms^{-1} starting from rest is nearly -

(A) 8.0 m (B) 4.1 m

(C) 16.4 m (D) 18.3 m

7. The pulley shown in the figure is mass less and frictionless. The motion of the bodies is frictionless. the acceleration produced in the system will be



(A) $\frac{F}{M}$ (B) $\frac{F}{m}$

(C) $\frac{F}{M+m}$ (D) $\frac{F}{M-m}$

8. In the above problem, the tension produced in the string will be -

(A) $\frac{Fm}{M}$ (B) $\frac{FM}{m}$

(C) $\frac{Fm}{M-m}$ (D) $\frac{Fm}{M+m}$

10. A block of mass 2 kg is lying on a floor. The coefficient of static friction is 0.54. What will be value of frictional force if the applied force is 2.8 N and g = 10 m/s^2 –

(A) 2.8 Newton (B) 8 Newton

(C) zero (D) 2 Newton

11. A block of mass 0.5 kg rests against a wall exerting a horizontal force of 15 N on the wall. If the coefficient of frictional between the wall and the block is 0.5 then the frictional force acting on the block will be

(A) 0.49 N (B) 4.90 N

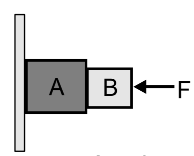
(C) 9.8 N (D) 49.9 N

12. A body of mass M is kept on a rough horizontal surface with coefficient of friction μ. A person tries to pull the body by applying a horizontal force, F but the body does not move. The force (F) on the body by the surface is

(A) F = mg (B) F = μMg

(C) F = Mg $\sqrt{1-u^2}$ (D) None of the above

13. All the surfaces shown in the figure are rough. The direction of friction on B due to A is



(A) zero (B) to the left

(C) upwards (D) downwards

14. A brick of mass 2kg just begins to slide down an inclined plane at an angle of 45º with the horizontal. The force of friction will be

(A) 19.6 cos 45º (B) 119.6 sin 45º

(C) 9.8 sin 45º (D) 9.8 cos 45º

15. A body is projected along a rough horizontal surface with a velocity 6 m/s. If the body comes to rest after travelling a distance 9m, the coefficient of sliding friction is (g = 10 m/s^2)

(A) 0.5 (B) 0.6

(C) 0.4 (D) 0.2

DYNAMICS OF CIRCULAR MOTION

Reference Frames

In order to study rest and motion, one needs to observe the position of bodies, which can be done only with respect to a frame of reference. The frame of reference consists of an observer, with a coordinate frame (Cartesian or, other wise) to measure position and clocks to measure time. Frame of references of two types.

Inertial Frame of Reference

Inertial frames are those which do not have any acceleration, i.e. either the frame at rest or it is moving with a uniform speed. In such frames we can directly apply Newton's laws and generate dynamic equations of the objects present in the frame.

Non-Inertial Frame of Reference

Non-inertial frames are accelerated reference frames and Newton's laws are not directly applicable in such frames, before application of Newton's Laws some modifications are required to solve the problem.

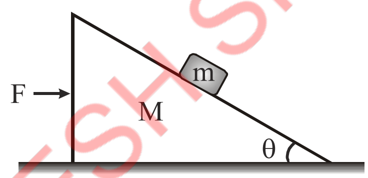
(i) Apply a pseudo force on an object if and only it is placed on another object (Non-Inertial frame) accelerating with respect to some inertial reference frame (i.e., earth).

(ii) The direction of pseudo force must be opposite to the direction of acceleration of the non-Inertial frame.

(iii) The magnitude of pseudo force is the product of mass of the body and acceleration of the non-inertial frame.

Example

All surfaces are smooth in adjoining figure. Find F such that block remains stationary with respect to the wedge.

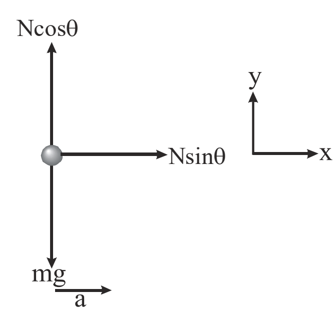


Solution

Acceleration of (block + wedge) $a = \dfrac{F}{(M+m)}$

Let us solve the Example by both the methods.

Method I : From inertial frame of reference (Ground) (apply real forces): w.r.t. ground block is moving with an frame of reference Frame of Reference acceleration a



$\Sigma F_y = 0 \Rightarrow N \cos\theta = mg$ ...(i)

and $\Sigma F_x = ma \Rightarrow N \sin\theta = ma$ ...(ii)

From equation (i) and (ii) i.e.,

$F = (M + m)a = (M + m)g \tan\theta$.

Method II : From non-inertial frame of reference (Wedge)

F.B.D. of block wrt. wedge (real forces + pseudo force). Block is stationary wrt. wedge.

$\Sigma F_y = 0 \Rightarrow \cos\theta = mg$ ...(iii)

$\Sigma F_x = 0 \Rightarrow N \sin\theta = ma$ ...(iv)

From equation (iii) and (iv), we will get the same result F = (M + m) g tanθ

Dynamics of uniform circular motion concept of centripetal force.

CENTRIPETAL FORCE

Centripetal force is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.

Centripetal acceleration $a = \frac{\upsilon^2}{r} = r\,\omega^2$

Where v is the linear velocity, is angular velocity of the body and *r* is the radius of the circular path.

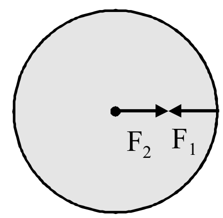
As F = m *a*, therefore,

centripetal force = mass x centripetal acceleration.

$F = m v^2/r = mr\,\omega^2$

CENTRIFUGAL FORCE

Centrifugal force is a force that arises when a body is moving actually along a circular path by virtue of tendency of the body to regain its natural straight line path. Centrifugal force can be regarded as the reaction of centripetal force. Centripetal force acts along the radius and away from the centre of the circle.



Centripetal force F_1 and Centrifugal force F_2.

PARTICLE APPLICATION OF CIRCULAR MOTION

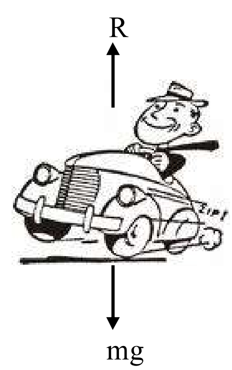
1. Motion of a car on a level road

Three forces act on the car:

(i) The weight of the car, **mg**

(ii) Normal reaction, **R**

(iii) Frictional force, **F**



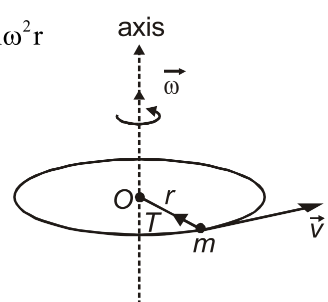
As there is no acceleration in the vertical direction

R–mg = 0; R = mg

The centripetal force required for circular motion is along the surface of the road , towards the center of the turn.

Neglecting gravity we get,

$$T = \text{Centripetal force} = \frac{mv^2}{r} = m\omega^2 r$$



Example:

A car is travelling at 30 km/h in a circle of radius 60 m. What is the minimum value of μ_s for the car to make the turn without skidding ?

Solution

The minimum μ_s should be that

$$\mu_S mg = \frac{mv^2}{r} \qquad \text{or} \qquad \mu_S = \frac{v^2}{rg}$$

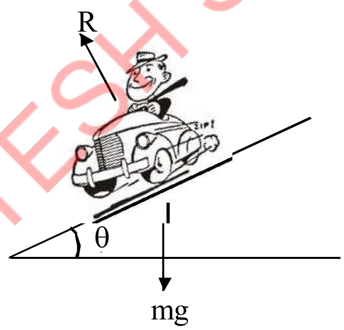
Hare $v = 30 \frac{km}{h} = \frac{30 \times 1000}{3600} = \frac{25}{3}$ m/s

$$\mu_S = \frac{25}{3} \times \frac{25}{3} \times \frac{1}{60 \times 10} = 0.115$$

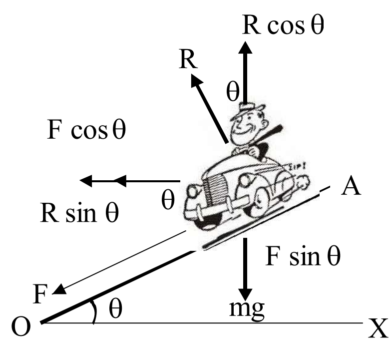
For all values of μ_S greater than or equal to the above value, the car can make the turn without skidding. If the speed of the car is high so that minimum μ_S is greater than the standard values (rubber tyres on dry concrete μ_S = 1 and on wet concrete μ_S = 0.7), then the car will skid.

2. Motion of a car on a banked road

The phenomenon of raising outer edge of the curved road above the inner edge is called banking of roads.



$\angle XOA = \theta$ =angle of banking



(i) mg (downwards).

(ii) R (perpendicular to OA).

(iii) Force of friction F (along AO).

(iv) R can be resolved into two rectangular components:

(v) $R\cos\theta$ along vertically upward direction

(vi) $R\sin\theta$ along the horizontal, towards the centre of the curved road.

(vii) F can also be resolved into two rectangular components:

(viii) $F\cos\theta$ along the horizontal, towards the centre of curved road.

(ix) F $\sin\theta$ along vertically downward direction.

$R \cos\theta = mg + F \sin\theta$ ...(i)

centripetal force required = mv^2/r

$R \sin\theta + F \cos\theta = \frac{mv^2}{r}$ ...(ii)

But $F \le \mu_s R$, where μ_s is coefficient of static friction between the banked road and the tyres. To obtain v_{max},

we put $F = \mu_s R$ from (i) & (ii)

$R \cos\theta = mg + \mu_s R \sin\theta$ ...(iii)

and $R \sin\theta + \mu_3 R \cos\theta = \frac{mv^2}{r}$ ...(iv)

From (iii), $R(\cos\theta - \mu_3 \sin\theta) = mg$

$$R = \frac{mg}{\cos\theta - \mu_3 \sin\theta}$$

From (iv),

$$R(\sin\theta + \mu_3 \cos\theta) = \frac{mv^2}{r} \qquad \text{...(v)}$$

Using (v).

$$\frac{mg(\sin\theta + \mu_3 \cos\theta)}{(\cos\theta - \mu_3 \sin\theta)} = \frac{mv^2}{r}$$

$$\therefore \quad v^2 = \frac{rg(\sin\theta + \mu_3 \cos\theta)}{(\cos\theta - \mu_3 \sin\theta)}$$

$$= \frac{rg\sin\theta(\tan\theta + \mu_3)}{\cos\theta(1 - \mu_3 \tan\theta)}$$

$$v = \left[\frac{rg(\mu_3 + \tan\theta)}{(1 - \mu_3 \tan\theta)}\right]^{1/2}$$ This is the max. velocity of vehicle on a banked road.

For a smooth surface $\mu = 0 \Rightarrow v = \sqrt{rg\tan\theta}$

For a horizontal surface, $\theta = 0 \Rightarrow v = \sqrt{\mu rg}$

Example

At what should a highway be banked for cars travelling at a speed of 100 km/h if the radius of the road is 400 m and no frictional forces are involved?

Solution

The banking should be done at an angle θ such that

$$\tan\theta = \frac{v^2}{rg} = \frac{\frac{250}{9} \times \frac{250}{9}}{400 \times 10}$$

or tan θ = $\frac{625}{81 \times 40}$ = 0.19

or θ = $\tan^{-1}$ 0.19 ≈ 0.19 radian

≈ 0.19 × 57.3º

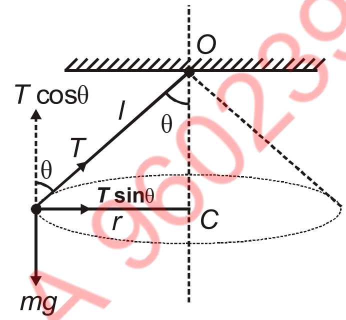
≈ 11º

3. (Considering gravity) Conical pendulum

$T\sin\theta = m\omega^2 r$ ...(i)

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ ...(ii)



(a) For θ to be 90º (*i.e.*, string to be horizontal)

T = ∞

∴ It is not possible.

(b) $T\sin\theta = m\omega^2 r = m\omega^2 l\sin\theta$

$\Rightarrow \quad T = m\omega^2 l$

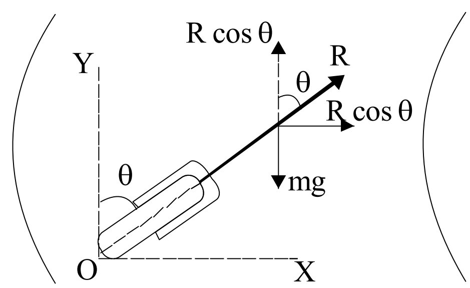
(c) Time period = $2\pi\sqrt{\frac{l\cos\theta}{g}}$

4. Bending of a cyclist

When a cyclist negotiates a curve, he bends slightly from his vertical position towards the inner side of the curve (see figure).

The various forces acting on the system (cycle of man) are :

(i) Weight (mg) of the system acts vertically downward through the centre of the gravity of the system.



(ii) Normal reaction (R) offered by the road to the system and acts at an angle θ with the vertical.

It is assumed that force of friction between the tyres of a bicycle and the surface is negligible.

Resolve R into two components:

(i) R cos θ which is equal and opposite to the weight (mg) of the system.

i.e. R cos θ = mg ...(1)

(ii) R sin θ, which is directed towards the centre of the circular path and provides the necessary centripetal force $\left(\frac{mv^2}{r}\right)$ to the system to remain in the circular path.

i.e. $R\sin\theta = \frac{mv^2}{r}$ ...(2)

Dividing (2) by (1), we get $\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$

$\tan\theta = \frac{v^2}{rg}$

or $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Clearly, θ depends upon v and r at a place.

For safe turn, θ should be small. θ will be small if v is small and r is large. Thus turning of the cycle should be at slow speed and along a track of larger radius.

5. An aeroplane making a turn

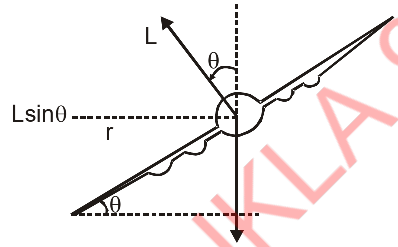
In order to makes a circular turn, a plane must roll at some angle θ in such a manner that the horizontal component of the lift force L provides the necessary centripetal force for circular motion. The vertical component of the lift force balances the weight of the plane.

$L \sin\theta = \frac{mv^2}{r}$

and $L \cos\theta = mg$

or the angle θ should be such that

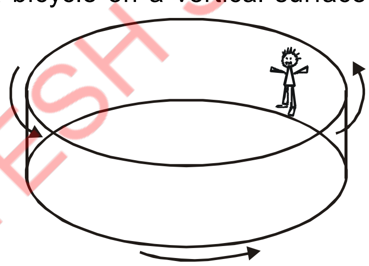
$\tan\theta = \frac{v^2}{rg}$



6. Death well and rotor

Example of uniform circular motion

In 'death well' a person drives a bicycle on a vertical surface of a large wooden well.



(a) A passenger on a 'rotor ride'

In 'death well' walls are at rest while person revolves.

In a rotor at a certain angular speed of rotor a person hangs resting against the wall without any floor.

In rotor person is at rest and the walls rotate.

In both these cases friction balances the weight of person while reaction provides the centripetal force necessary for circular motion i.e.

Force of fiction F_S = mg and Normal reaction $F_N = \frac{mv^2}{r}$

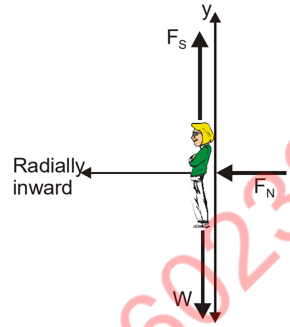
so $\frac{F_N}{F_S} = \frac{v^2}{rg}$, i.e., $v = \sqrt{\frac{rgF_N}{F_S}}$

Now for v to be minimum F_S must be maximum, i.e., $v_{min} = \sqrt{\frac{rg}{\mu}}$

[as $F_{S\,max} = \mu F_N$]

Example

1. A 62 kg woman is a passenger in a "rotor ride" at an amusement park. A drum of radius 5.0 m is spun with an angular velocity of 25 rpm. The woman is pressed against the wall of the rotating drum as shown in fig. (a) Calculate the normal force of the drum of the woman (the centripetal force that prevents her from leaving her circular path). (b) While the drum rotates, the floor is lowered. A vertical static friction force supports the woman's weight. What must the coefficient of friction be to support her weight ?



(b) A force diagram for the person

Solution.

Normal force exerted by the drum on woman towards the centre

$$F_N = ma_c = m\omega^2 r = 62\text{ kg} \times \left(25\frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{rad}}{1\text{rev}} \times \frac{1\text{min}}{60\text{s}}\right)^2 \times 5\text{m} = 2100\text{ N}$$

(b) $\mu F_N = F = mg$ ....(2) dividing eqn. (2) be eq. (1)

$$\mu = \frac{g}{\omega^2 r} = \left(\frac{60}{2\pi \times 25}\right)^2 \times \frac{10}{5} = 0.292$$

2. A 1.1 kg block slides on a horizontal frictionless surface in a circular path at the end of a 0.50 m long string. (a) Calculate the block's speed if the tension in the string in 86 N. (b) By what percent does the tension change if the block speed decreases by 10 percent?

Solution

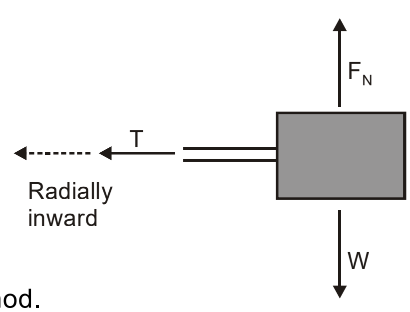
(a) Force diagram for the block is shown in fig. The upward normal force balances the block's weight. The tension force of the string on the block provides the centripetal force that keeps the block moving in a circle. Newton's second law for forces along the radial direction is

$$\sum F \text{ (in radial direction)} = T = \frac{mv^2}{r}, \quad \text{or} \quad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{(80\text{N})(0.50\text{m})}{1.2\text{ kg}}} = 5.0\text{ m/s}$$

(b) A 10 percent reduction in the speed results in a speed v' = 5.4 m/s. The new tension is

$$\frac{T'}{T} = \frac{(mv'^2/r)}{(mv^2/r)} = \left(\frac{v'}{v}\right)^2 = \left(\frac{0.90}{v}\right)^2 = 0.81$$

Thus, $\frac{T'}{T} = \frac{70\text{N}}{86\text{N}} = 0.81$



percentage reduction in the tension is about 19%.

The same result is obtained using a proportionality method.

$$\frac{T'}{T} = \frac{(mv'^2/r)}{(mv^2/r)} = \left(\frac{v'}{v}\right)^2 = \left(\frac{0.90}{v}\right)^2 = 0.81$$

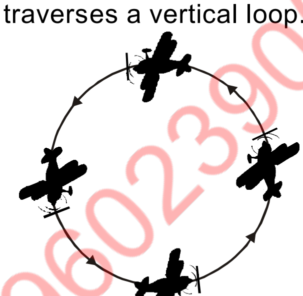
7. Looping the loop

This is the best example of non uniform circular motion in vertical plane.

For looping the pilot of the plane puts off the engine at lowest point and traverses a vertical loop. (with variable velocity)

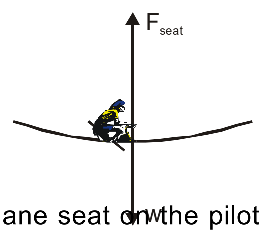
Example

An aeroplane moves at 64 m/s in a vertical loop of radius 120 m, as shown in fig. Calculate the force of the plane's seat on 172 kg pilot while passing through the bottom part of the loop.



Solution.

Two forces act on the pilot his downward weigth force w and the upward force of the aeroplane's seat F_{seat}. Because the pilot moves in a circular path, these forces along the radial direction must, according to Newton's seconds law ($\sum$ F = ma), equal the pilot's mass times his centripetal acceleration, where $a_c = v^2/r$.



We find that $\sum$ F (in radial direction) = $F_{seat} - w = \dfrac{mv^2}{r}$

Remember that force pointing towards the center of the circle (F_{seat}) is positive & those pointing away from the center (w) are negative.

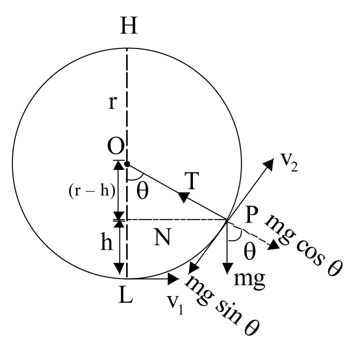
Substituting ω = mg and rearranging, we find that the force of the aeroplane seat on the pilot is

$$F_{seat} = m\left(\frac{v^2}{r} + g\right) = 72\text{ kg}\left[\frac{64(m/s)^2}{120m} + 9.8m/s^2\right] = 72\text{ kg }(34.1\text{ m/s}^2 + 9.8\text{m/s}^2) = 3160.8\text{ N}$$

The pilot in this example feels very heavy. To keep him in the circular path, the seat must push the pilot upwards with a force of 3160 N, 4.5 times his normal weight. He experiences an acceleration of 4.5 g, that is, 4.5 times the acceleration of gravity.

Motion in a Vertical Circle

Let us consider a stone of mass m tied to one end of the string and moving in a vertical circle of radius r (which is equal to the length of the string) as shown in figure. When it is at the highest point H, it should fall vertically downward because of gravity but on the contrary, it does not fall. The velocity of the stone is minimum at the highest point and maximum at the lowest point.



Let v_1 be the velocity of the stone at the lowest point L of the circle. Suppose at any instant of time, stone reaches point P. Let the velocity of stone at P = v_2.

Height through which the stone is raised, when it goes from point L to point P = h

Now, K.E. of stone of mass m at $L = \dfrac{1}{2}mv_1^2$

P.E. of the stone at L = 0

$\therefore$ Total energy of the stone at $L = \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_1^2$

K.E. of stone at $P = \frac{1}{2}mv_2^2$

P.E. of stone at P = mgh

$\therefore$ Total energy of the stone at $P = \frac{1}{2}mv_2^2 + mgh$

According to the law of conservation of energy,

$$\frac{1}{2}mv_2^2 + mgh = \frac{1}{2}mv_1^2$$

or $$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - mgh$$

$$v_2^2 = v_1^2 - 2gh$$

As is evident from equation (1), v_2 is always less than v_1.

Tension in the string

Let T be the tension in the string.

Therefore, various forces acting on the stone or particle at P are :

(i) Tension **T** in the string acting along PO

(ii) Weight **mg** of the particle acting vertically downward.

Resolve **mg** into two components :

(i) mg cos θ which acts in a direction opposite to the direction of tension T.

(ii) mg sin θ which acts along the tangent at P.

$\therefore$ Net force acting on the stone or particle towards the centre of the circle = T – mg cos θ

This force provides the necessary centripetal force $\frac{mv_2^2}{r}$ to the stone to move in the circle.

i.e., $$T - mg\cos\theta = \frac{mv_2^2}{r}$$

or $$T = \frac{mv_2^2}{r} + mg\cos\theta \qquad ...(2)$$

Substituting the value of v_2 from equation (1) in (2), we get

$$T = \frac{m}{r}[v_1^2 - 2gh] + mg\cos\theta$$

Now from ΔPON,

$$\cos\theta = \frac{r-h}{r}$$

$$T = \frac{m}{r}[v_1^2 - 2gh] + mg\left(\frac{r-h}{r}\right) \qquad ...(3)$$

Tension at Lowest point L.

At lowest point, h = 0

$$\therefore \quad T_L = \frac{mv_1^2}{r} + mg = \frac{m}{r}(v_1^2 + gr)$$

Tension at highest point H

At highest point, h = 2r

Therefore, from equation (3), we have

$$T_H = \frac{m}{r}[v_1^2 - 2g(2r)] + mg\left(\frac{r-2r}{r}\right) = \frac{m}{r}(v_1^2 - 4gr) - mg$$

$$T_H = \frac{m}{r}[v_1^2 - 4gr - gr]$$

$$T_H = \frac{m}{r}[v_1^2 - 5gr - gr]$$

Difference in tension at the lowest and highest point.

$$T_L - T_H = \frac{m}{r}(v_1^2 + gr) - \frac{m}{r}(v_1^2 - 5gr) = \frac{m}{r}(6gr)$$

$$\therefore \quad T_L = T_H = 6mg$$

Minimum speed at the lowest point for looping the loop.

In order to keep the body moving in a circular path, the tension in the string must remain positive even at the highest point i.e., $T_H \geq 0$

$$\therefore \quad \frac{m}{r}(v_1^2 - 5gr) \geq 0$$

or $\quad v_1^2 \geq 5gr$

or $\quad v_1 \geq \sqrt{5gr}$

Hence, the minimum speed which the body (say a motor cycle) must possess at the lowest point so that it may go round the vertical circle is given by

$$v_1 = \sqrt{5gr}$$

To find the speed of the particle or body at the highest point of the vertical circle, substitute $v_1 = \sqrt{5gr}$ and h = 2r in equation (1), we get

$$v_2^2 = 5gr - 2g \times 2r = 5gr - 4gr = gr$$

$$\therefore \quad v_2 = \sqrt{gr}$$

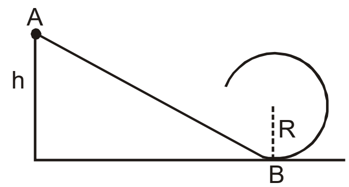
This is the velocity of the body at the highest point for looping the loop.

Application of motion in a vertical circle

1. Water in a bucket does not spill if it is rotated in a vertical circle.
2. A motor cyclist in a circus drives the motor cycle along a vertical circle in a cage.
3. A pilot loop in a loop without falling at the top of the loop.

SOLVED EXAMPLE

Example 1: A ball is released from height h as shown in fig. Find the condition for the particle to complete the circular path.



Solution. According to law of conservation of energy (K.E. + P.E) at A = (K.E. + P.E) at B

$\Rightarrow \quad 0 + mgh = \frac{1}{2} mv^2 + 0$

$\Rightarrow \quad v = \sqrt{2gh}$

But velocity at the lowest point of circle,

$v \geq \sqrt{5gR} \Rightarrow \sqrt{2gh} \geq \sqrt{5gR} \Rightarrow h \geq \frac{5R}{2}$

Example 2: A body weighing 0.4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1.2 m, find the tension in the string, when the body is (a) at the top of the circle (b) at the bottom of the circle. Given : g = 9.8 ms^{-2} and π = 1.2 m

Solution. Mass m = 0.4 kg time period = $\frac{1}{2}$ second and radius, r =1.2 m

Angular velocity, $\omega = \frac{2\pi}{1/2} = 4\pi$ rad s^{-1} = 12.56 rad s^{-1}

(a) At the top of the circle, $T = \frac{mv^2}{r} - mg = mr\omega^2 - mg = m\,(r\omega^2 - g)$

= 0.4 (1.2 × 12.56 × 12.56 – 9.8) N = 71.8 N

(b) At the lowest point, T = m($r\omega^2$ + g) = 79.64 m

Example 3: In a circus a motorcyclist moves in vertical loop inside a 'death well' (a hollow spherical chamber with holes, so that the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when the is at the uppermost point, with no support from below. What is the minimum speed required to perform a vertical loop if the radius of the chamber is 25 m?

Solution. When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards. These two forces are balanced by the outward centrifugal force acting on him.

$\therefore \quad R + mg = \frac{mv^2}{r}$ ....(i) r = radius of the circle

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (i), when R = 0

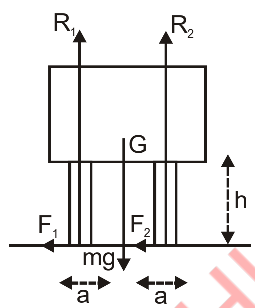
$$\therefore \quad mg = \frac{mv^2_{min}}{r} \quad \text{or} \quad v^2_{min} = gr$$

$$\text{or} \quad v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25}\ ms^{-1} = 15.65\ ms^{-1}$$

So, the minimum speed at the top required to perform a vertical loop is 15.65 ms^{-1}.

6.7 Condition of overturning

Here, we shell find the condition for the car to overturn. Let the distance between the centres of wheels of the car be 2a and the centre of gravity be h meters above the ground (road). The different forces acting on the car are shown in the fig.



- The weight mg of the car acts downwards through centre of gravity G.
- The normal reactions of the ground R_1 and R_2 on the inner and outer wheels respectively. These act vertically upwards.
- Let force of friction $F_1 + F_2$ between wheels and ground towards the centre of the turn.

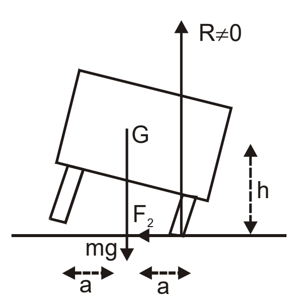
Let the radius of circular path be **r** and the speed of the car be **v**.

Since there is no vertical motion, equating the vertical forces, we have

$$R_1 + R_2 = mg \qquad ....(1)$$

The horizontal force = centripetal force for motion in a circle

$$\text{So,} \quad F = F_1 + F_2 = \frac{mv^2}{r} \qquad ....(2)$$



Taking moments about the centre of mass G.

$$(F_1 + F_2)\ h = R_1 a = R_2 a$$

$$\therefore F_1 + F_2 = (R_2 - R_1) = \frac{a}{h} \qquad ...(3)$$

Combining this with equation (2) to eliminate $F_1 + F_2$ gives

$$R_2 - R_1 = \frac{hmv^2}{ar} \qquad ...(4)$$

We now have two simultaneous equations, (1) and (4), for R_1 and R_2. Solving these by adding and subtracting, we find that

$2R_1 = mg - \frac{hmv^2}{ar}$ and $2R_2 = mg + \frac{hmv^2}{ar}$

From these expressions it is clear.

Inner wheels will leave the ground when R_1 will become zero and the car begins to overturn,

i..e, $mg = \frac{hmv^2}{ar}$

So the limiting speed is given by $v^2 = \frac{gra}{h}$ as required.

EXAMPLE

1. The radius of curvature of a railway line at a place when the train is moving with a speed of 36 kmh $^{-1}$ is 1000 m, the distance between the two rails being 1.5 metre. Calculate the elevation of the outer rail above the inner rail so that there may be no side pressure on the rails.

Solution :

Velocity, v = 36 km h^{-1} = $\frac{36 \times 1000}{3600}$ ms^{-1} = 10 ms^{-1}

radius, r = 1000 m; $\tan\theta$ = $\frac{v^2}{rg}$ = 1000×9.8 = $\frac{1}{9.8}$

Let h be the height through which outer rail is raised. Let ℓ be the distance between the two rails.

Then, $\tan\theta = \frac{h}{\ell}$ [$\because$ θ is very small]

or $h = \ell \tan\theta$

$h = 1.5 \times \frac{1}{98}$ m = 0.0153 m [$\because$ ℓ = 1.5 m]

2. An aircraft executes a horizontal loop oat a speed of 720 km h^{-1} with its wing banked at 15º. Calculate the radius of the loop.

Solution :

Speed , v = 720 km h^{-1} = $\frac{720 \times 1000}{3600}$ ms^{-1} = 200 ms^{-1} and $\tan\theta = \tan 15^\circ = 0.2679$

$\tan\theta = \frac{v^2}{rg}$ or $r = \frac{v^2}{g\tan\theta} = \frac{200 \times 200}{9.8 \times 0.2679}$ m

= 1523.7m = 15.24 km,

3. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km h^{-1}. The mass of the train is 10^6 kg. What provides the centripetal force required for this propose? The engine or the rails? The outer of inner rails ? Which rail will wear out faster, the outer or the inner rail ? What is the angle of banking required to prevent wearing out of the rails?

Solution:

r = 30 m, v = 54 km h^{-1} = $\frac{54\times5}{18}$ ms^{-1} = 15 ms^{-1} m = 10^6 kg, θ = ?

(i) The centripetal force is provided by the lateral thrust by the outer rail on the flanges of the wheel of the train. The train causes an equal and opposite thrust on the outer rail (Newton's third law of motion).

Thus, the outer rails wear out faster.

(ii) tan θ = $\frac{v^2}{rg}$ = $\frac{15\times15}{60\times9.8}$ = 0.7653

$\therefore$ $\theta = \tan^{-1}(0.7653) = 37.43^\circ$

4. A circular race tack of radius 300 m is banked at an angle of 150°. If the coefficient of friction between the wheels of a race car and the road is 0.2, what is the (a) optimum speed of the race car to avoid wear and tear of tyres, and the (b) maximum permissible speed to avoid slipping?

Solution:

(a) on a banked road, the horizontal component of the normal reaction and the friction force contribute to provide centripetal force to keep the car moving o a circular turn without slipping. At the optimum speed, the component of the normal reaction is enough to provide the required centripetal force. In this case, the frictional force is not required. The optimum speed is given by

$v_0 = (rg \tan\theta)^{1/2} = (300 \times 9.8 \tan 15^\circ)^{1/2}$ ms^{-1} = 28.1 ms^{-1}

(b) The maximum permissible speed if given by

$v_{max} = \left(\frac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)^{1/2}$ Substituting values and simplifying, we get v_{max} = 38.1 ms^{-1}.

POINTS TO BE REMEMBER

☞ **Uniform motion in a circle -**

Angular velocity $\omega = \frac{d\theta}{dt} = 2\pi n = \frac{2\pi}{T}$

Linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$

v = ωr when $\vec{\omega}$ and $\vec{r}$ are perpendicular to each other.

Centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r = \omega v = 4\pi^2 n^2 r$

☞ **Equations of motion -**

For constant angular acceleration -

(i) $\omega = \omega_0 + \alpha t$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

☞ **Motion of a car on a plane circular road -**

For motion without skidding

$$\frac{Mv_{max}^2}{r} = \mu M_g, v_{max}\sqrt{\mu rg}$$

☞ **Motion on a banked road -** Angle of banking = θ

$$\tan\theta = \frac{h}{b}$$

Maximum safe speed at the bend $v_{max} = \left[\frac{rg(\mu + \tan\theta)}{1-(\mu\tan\theta)}\right]^{1/2}$

If friction is negligible $v_{max} = \sqrt{rg\ \tan\theta}$ = $\sqrt{\frac{rhg}{b}}$

and $\tan\theta = \frac{v^2\ max}{rg}$

☞ **Motion of cyclist on a curve -**

In equilibrium angle with vertical is θ then $\tan\theta = \frac{v^2}{rg}$

Maximum safe speed = v_{max} = $\sqrt{\mu rg}$

☞ **Motion in a vertical circle (particle tied to string) -**

At the top position - Tension $T_A = m\left(\frac{v_A^2}{r} - g\right)$

For T_A = 0, critical speed $= \sqrt{gr}$

At the bottom - Tension $T_B = m\left(\frac{v_B^2}{r} + g\right)$

For completing the circular motion minimum speed at the bottom $v_B = \sqrt{5gr},$

Tension $T_B = 6mg$

☞ **Conical pendulum (Motion in a horizontal circle)**

Tension is string $= \frac{mg\ell}{(\ell^2 - r^2)^{1/2}}$

Angular velocity $= \sqrt{\frac{g}{\ell\cos\theta}}$

Periodic time $= 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$ $= 2\pi\sqrt{\frac{r}{g\tan\theta}}$

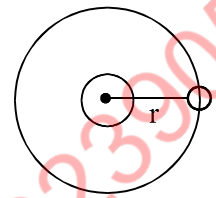
SOLVED EXAMPLE

Example 1. The circular motion of a particle with constant speed is

(A) Periodic but not simple harmonic (B) Simple harmonic but not periodic

(C) Periodic and simple harmonic (D) Neither periodic nor simple harmonic

Solution. **(A)**



In circular motion of a particle with constant speed, particle repeats its motion after a regular interval of time but does not oscillate about a fixed point. So, motion of particle is periodic but not simple harmonic.

Example 2. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill when the bucket is at the highest position

(A) 4 m/sec (B) 6.25 m/sec

(C) 16 m/sec (D) None of the above

Solution. **(A)**

Since water does not fall down, therefore the velocity of revolution should be just sufficient to provide centripetal acceleration at the top of vertical circle. So,

$v = \sqrt{(g\,r)} = \sqrt{\{10 \times (1.6)\}} = \sqrt{(16)}$ = 4 m/sec.

Example 3. Explain why curved roads are generally banked?

Solution. The force of friction between the road and the tyres provides the necessary centripetal force to the vehicle to negotiate the flat curved roads. If the force of friction is insufficient to provide the necessary centripetal force to the vehicle, it will skid and go off the road. To avoid this, the road is banked i.e. the outer edge of the road is raised a little above the inner edge.

EXERCISE

1. A car moves along a horizontal circular road of radius r with velocity v. The coefficient of friction between the wheels and the road is μ. Which of the following statements is not true?

(A) The car will slip if $v > \sqrt{\mu r g}$

(B) The car will slip if $\mu < \frac{v^2}{rg}$

(C) The car will slip if $r > \frac{v^2}{\mu g}$

(D) The car will slip at a lower speed, along with some acceleration, than if it moves at constant speed

2. The driver of a car travelling at velocity v suddenly sees a broad wall in front of him at a distance d. He should

(A) Brake sharply (B) Turn sharply

(C) (A) and (B) both (D) None of the above

3. A cyclist riding the bicycle at a speed of $14\sqrt{3}\ ms^{-1}$ takes a turn around a circular road of radius $20\sqrt{3}$ m without skidding. (Given g = 9.8 ms^{-2}), what is his inclination to the vertical

(A) 30° (B) 90°

(C) 45° (D) 60°

4. Consider the following statements

Assertion (A) : A cyclist always bends inwards while negotiating a curve

Reason (R) : By bending he lowers his centre of gravity of these statements :

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

5. A racing car of $1000\ kg$ moves round a banked track at a constant speed of $108\ km/hr$. Assuming the total reaction at the wheels is normal to the track and the horizontal radius of the track is $100\ m$, calculate the angle of inclination of the track to the horizontal (take g = 10 m/s^2)

(A) 12^o (B) 27^o

(C) 42^o (D) 65^o

6. In the above question, what is the reaction at the wheels?

(A) 13450 N (B) 26900 N

(C) 6725 N (D) 40350 N

7. A car is moving with a speed v on a road inclined at an angle θ in a circular arc of radius r, the minimum coefficient of friction, so that the car does not slip away

(A) $\frac{v^2}{rg} = \mu \tan\theta$ (B) $\mu = v^2/rg$

(C) $\frac{v^2\cos\theta - rg\sin\theta}{rg\cos\theta + v^2\sin\theta}$ (D) $\frac{v^2\cos\theta - rg\sin\theta}{rg\cos\theta - v^2\sin\theta}$

8. A weightless thread can bear tension upto $3.7\ kg\ wt$. A stone of mass $500\ gms$ is tied to it and revolved in a circular path of radius $4m$ in a vertical plane. If g = 10 m/s^2, then the maximum angular velocity of the stone will be

(A) 4 radians/sec (B) 16 radians / sec

(C) $\sqrt{21}$ radians / sec (D) 2 radius / sec

9. A $2kg$ stone at the end of a string $1m$ long is whirled in a vertical circle at a constant speed. The speed of the stone is $4m/\sec$. The tension in the string will be $52\ N$, when the stone is

(A) At the top of the circle (B) At the bottom of the circle

(C) Halfway down (D) None of the above

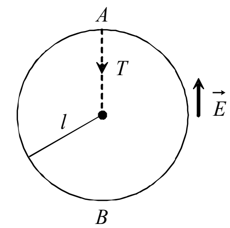
10. A particle of mass M carries a charge +Q. It is attached to a string of length l and is whirled in a vertical circle in electric field $\vec{E}$ directed upwards; what should be the minimum speed of the particle at A so that it loops the loop

(A) $[(g + QE/M)l]^{1/2}$

(B) $[(g - QE/M)l]^{1/2}$

(C) $[(Mg + QE)l]^{1/2}$

(D) $[(Mg - QE)l]^{1/2}$



11. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$

(C) $\sqrt{u^2 - gl}$ (D) $\sqrt{2(u^2 - gL)}$

Important Competition Tips

1. In the absence of the force, a body moves along a straight line path.
2. A system or a body is said to be in equilibrium, when the net force acting on it is zero.
3. If a **number of forces** $\vec{F}_1, \vec{F}_2, \vec{F}_3, ...$ act on the body, then it is in equilibrium when $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = \vec{0}$
4. A body in equilibrium cannot change the direction of motion.
5. If a body moves along a curved path, then it is certainly acted upon by a force.
6. Forces in nature always occur in pairs.
7. $1\ N = 10^5\ dyne.$
8. Gravitational units of force are *gf* (or *gwt*) in *CGS* system and *kgf* (or *kgwt*) in *SI*.
9. $1\ gf = 980\ dyne$ and $1\ kgf = 9.8\ N$
10. The beam balance compares masses.
11. Acceleration of a horse-cart system is $a = \frac{H - F}{M + m}$ where H = Horizontal component of reaction; F = force of friction; M = mass of horse; m = mass of cart.
12. The weight of the body measured by the spring balance in a lift is equal to the apparent weight.
13. Apparent weight of a freely falling body = ZERO, (state of weightlessness).
14. If the person climbs up along the rope with acceleration *a*, then tension in the rope will be $m(g+a)$
15. If the person climbs down along the rope with acceleration, then tension in the rope will be $m(g - a)$
16. When the person climbs up or down with uniform speed, tension in the string will be *mg*.
17. For an isolated system (on which no external force acts), the total momentum remains conserved (Law of conservation of momentum).
18. The change in momentum of a body depends on the magnitude and direction of the applied force and the period of time over which it is applied *i.e.* it depends on its impulse.
19. Guns recoil when fired, because of the law of conservation of momentum. The positive momentum gained by the bullet is equal to negative recoil momentum of the gun and so the total momentum before and after the firing of the gun is zero.
20. Recoil velocity of the gun is $\vec{V} = \frac{-m}{M}\vec{v}$
21. where *m* = mass of bullet, M = mass of gun and $\vec{v}$ = muzzle velocity of bullet.

22. The rocket pushes itself forwards by pushing the jet of exhaust gases backwards.

23. Upthrust on the rocket = $u \times \frac{dm}{dt}$ .where u = velocity of escaping gases relative to rocket and $\frac{dm}{dt}$ = rate of consumption of fuel.

24. Initial thrust on rocket = $m(g + a)$, where a is the acceleration of the rocket.

25. Impulse, $\vec{I} = \vec{F} \times \Delta t =$ change in momentum

26. Unit of impulse is N-s.

27. Action and reaction forces never act on the same body. They act on different bodies. If they act on the same body, the resultant force on the body will be zero i.e., the body will be in equilibrium.

28. Action and reaction forces are equal in magnitude but opposite in direction.

29. Action and reaction forces act along the line joining the centres of two bodies.

30. Newton's third law is applicable whether the bodies are at rest or in motion.

31. The non-inertial character of the earth is evident from the fact that a falling object does not fall straight down but slightly deflects to the east.